Math 205 Assignment #7 Solutions /30

11.1

#24) a) $y'' - y = 0$
b) $x^2y'' + 2xy' - 2y = 0$
c) $x^2y'' - 6y = 0$

use inspection and check to see if (i) $y = e^x$ (ii) $y = x^3$ (iii) $y = e^{-x}$ (iv) $y = x^{-2}$

a) $(e^x)'' - e^x = 0 \therefore$ (i)
$(e^{-x})'' = e^{-x} \therefore$ (iii)

b) $x^2(6x) + 2x(3x^2) - x^3 \neq 0$
$x^2(+6x^{-4}) + 2x(-2x^{-3}) - 2x^{-2} = 0 \therefore$ (iv)

c) $x^2y'' - 6y = 0$ x^3 works! $\therefore$ (ii)

#28) let $y = A + Be^{-2t}$

a) $\frac{dy}{dt} = 100 - 2y$ $\therefore$ plug in to get $-2Be^{-2t} = 100 - 2y$

$\therefore -2Be^{-2t} = 100 - (A + Be^{-2t}) = 100 - A - Be^{-2t}$

$\therefore -Be^{-2t} = 100 - A$ (2)

to be more specific you will need an IC

b) $\int \frac{dy}{50-y} = \int 2\,dt$ $\therefore -\ln \frac{50-y}{50-y(0)} = 2t$ $\therefore \frac{50-y}{(50-y(0))} = e^{-2t}$

$\therefore 50 - y(t) = (50 - y(0))e^{-2t}$ $\therefore y(t) = 50 - (50 - y(0))e^{-2t}$ (3)

$y(0) = 85$ $\therefore y(t) = 50 - (50-85)e^{-2t} = \underline{50 + 35e^{-2t}}$

11.2 #4)

a) $y'(0) = 1$ ∴ (V) is the only one that could work

b) $y' = 1$ when $y = 0$ ∴ (I) works

c) $y' = \cos(4-y) = 1$ when $y = 4$ ∴ (V)

d) $y' = y(4-y) = 0$ when $y = 4$ ∴ (II)

e) $y' = y(3-y) = -4$ when $y = 4$ ∴ (VI)

f) $y' = x(3-x) = -1$ when $x = 4$ ∴ (IV)

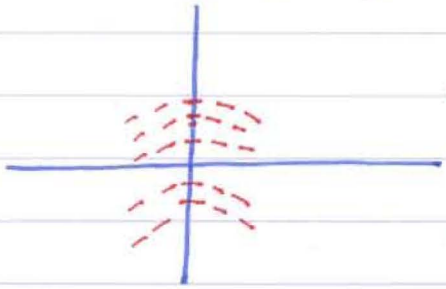
* students may use the online slope-field tool (or just calculate)

#10) Students may use the slope-field tool for this

$y' > 0$, $x < 0$

$y' < 0$, $x > 0$

$y' = 0$, $x = 0$



} something like this satisfies the 3 conditions

11.3 #6)

a) $\frac{dy}{dx} = x^3$, $y(0) = 0$ ∴ $y_{i+1} = y_i + F(x_i, y_i)\,h$

∴ $y_{i+1} = y_i + x_i^3\,h$

x	y	y'
0	0	0
0.1	0	0.001
0.2	0	0.008
0.3	0	0.027
0.4	0	.
⋮	0.01	.
1	⋮	⋮

(2) see attached spreadsheet

there a pretty small at first

b) $\frac{dy}{dx} = x^3 \Rightarrow dy = x^3\,dx$

$\therefore y = \frac{1}{4}x^4 + C$

(2)

$y(0) = 0 \Rightarrow C = 0$

$\therefore y = \frac{1}{4}x^4 + 0 = \frac{1}{4}x^4$

c) Any reasonable attempt at a slope field is ok here – don't be too picky!

(1)

$$\frac{dy}{dx}=\sin(xy)$$

h=0.2

x	y	y'
1	1	0.841471
1.2	1.168294	0.98578
1.4	1.36545	0.942476
1.6	1.553945	0.609381
1.8	1.675822	0.124788
2	**1.700779**	**-0.25705**
2.2	1.64937	-0.468
2.4	1.555771	-0.55824

h=0.1

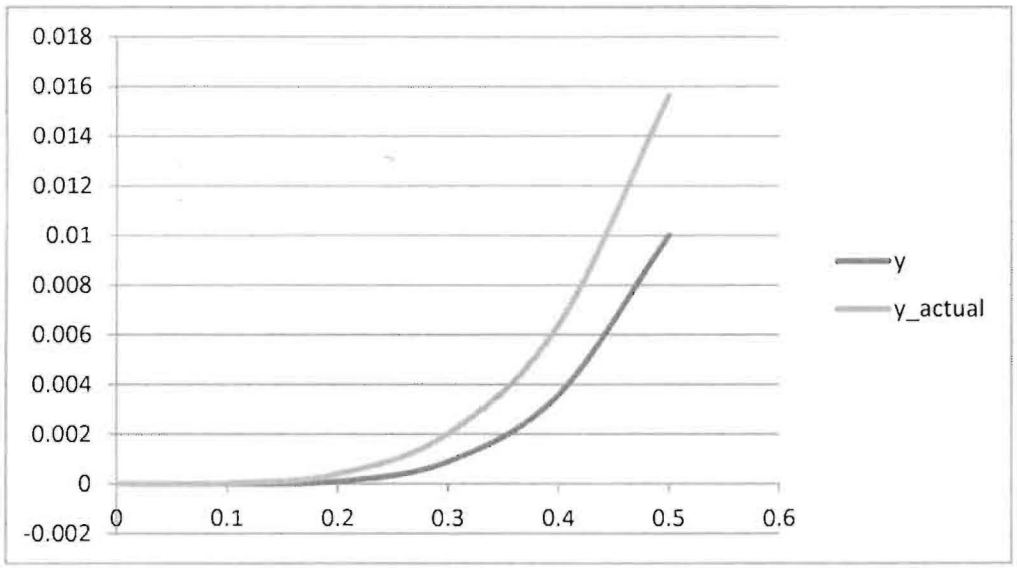
x	y	y'
1	1	0.841471
1.1	1.084147	0.929318
1.2	1.177079	0.987496
1.3	1.275829	0.99615
1.4	1.375444	0.937707
1.5	1.469214	0.806242
1.6	1.549838	0.614578
1.7	1.611296	0.391618
1.8	1.650458	0.169939
1.9	1.667452	-0.02656
2	**1.664796**	**-0.18689**
2.1	1.646106	-0.31004
2.2	1.615103	-0.40011
2.3	1.575092	-0.46277

h=0.05

x	y	y'
1	1	0.841471
1.05	1.042074	0.888551
1.1	1.086501	0.930271
1.15	1.133015	0.964348
1.2	1.181232	0.98827
1.25	1.230646	0.999472
1.3	1.280619	0.995584
1.35	1.330398	0.97474
1.4	1.379135	0.935899
1.45	1.42593	0.879111
1.5	1.469886	0.805645
1.55	1.510168	0.717936
1.6	1.546065	0.61933
1.65	1.577031	0.513699
1.7	1.602716	0.404996
1.75	1.622966	0.296859
1.8	1.637809	0.19233
1.85	1.647426	0.093717
1.9	1.652112	0.002581
1.95	1.652241	-0.08019
2	**1.648231**	**-0.15425**
2.05	1.640518	-0.21966
2.1	1.629535	-0.27677
2.15	1.615697	-0.32608

$$\frac{dy}{dx} = x\text{^}3$$

3 for table
2 for the rest

h 0.1

x	y	y'	y_actual	error
0	0	0	0	#DIV/0!
0.1	0.00	0.001	0.000025	39
0.2	0.00	0.008	0.0004	19
0.3	0.00	0.027	0.002025	12.33333
0.4	0.00	0.064	0.0064	9
0.5	0.01	0.125	0.015625	7
0.6	0.02	0.216	0.0324	
0.7	0.04	0.343	0.060025	
0.8	0.08	0.512	0.1024	
0.9	0.13	0.729	0.164025	
1	0.20	1	0.25