

Math 205 Assign # 8

Set up 2

14) $d(t)$ = the death rate $b(t)$ = birth rate = 140
 $P(t)$ = world population P_0 = population at 2010
 $= 6.9$ billion

$\therefore \frac{dP(t)}{dt}$ = rate of change of population $t_0 = 2010$

The population will change because of 2 competing rates: birth + death

model +

$$\therefore \frac{dP}{dt} = (b(t) - d(t)) \rightarrow \text{this makes sense! explain}$$

5) a) assume $d(t)$ is linear $d(0) = 57$ $d(30) = 80$ (million)

$$\therefore d(t) = \frac{180-57}{30}t + 57 = dt + d_0 \quad d = 0.77 \quad d_0 = 57$$

$$\therefore \frac{dP}{dt} = (b(t) - d(t)) = (140 - dt - d_0)$$

$$\therefore \frac{dP}{dt} = 83 - 0.77t ; t=0 \text{ at } 2010, P_0 = 6.9 \text{ billion}$$

b) Solve by separation $\therefore \int dP = \int (83 - 0.77t) = 83t - 0.385t^2 + C$

Solve $\therefore P(0) = 6.9$ billion = 83 million(0) - 0 + C $\therefore C = 6.9$ billion

3) $\therefore P(t) = (83 \text{ million})t - (0.385 \text{ million})t^2 + 6.9 \text{ billion}$
 $P(t) = (8.3 \times 10^7)t - 0.385t^2 - (3.85 \times 10^5)t^2 + 6.9 \times 10^9$

c) in 2050, $t = 40$ years

$$\therefore P(40) = 9.6 \times 10^9 \text{ or } \underline{\underline{9.6 \text{ billion people}}}$$

#24) $L = \text{total number of people who want to see a movie}$
 $N(t) = \text{the # people who have seen the movie, } N(0) = 0$!
 $\therefore L - N(t) = \# \text{ of people who have not yet seen the movie}$

$\frac{dN}{dt} = \text{rate of people who got the movie and equals (proportional to) the #}$
 $= \text{who have not seen it yet}$

5 } $\therefore \frac{dN}{dt} = \alpha(L - N(t))$; $\alpha = \text{proportionality constant}$

this makes sense because
 the rate should drop as more people
 have already seen the movie

Solve via $\int_{L-N(t)}^L \frac{dN}{dt} dt = \int_{t=0}^t dt = \alpha t$

3 } $\therefore -\ln(L - N(t)) \Big|_0^t = \alpha t \quad \therefore \ln(L - N(t)) \Big|_0^t = -\alpha t$

$\therefore \ln \left(\frac{L - N(t)}{L - N_0} \right) = -\alpha t, N_0 = 0$ (no one has seen it yet!)

$\therefore \frac{L - N(t)}{L} = e^{-\alpha t} \quad \therefore N(t) = L(1 - e^{-\alpha t})$

#29) Note: Students should review pages 630 - 633 carefully!

$P = \text{total oil production}$ $\frac{dP}{dt} = \text{annual production}$

a) $1993: \frac{dP}{dt} = 22 \text{ billion barrels/a}$

$2008: \frac{dP}{dt} = 26.9 \text{ bn/a}$

b) Just divide to get relative growth rate, $\frac{1}{P} \frac{dP}{dt}$

$\therefore 1993: \frac{1}{P} \frac{dP}{dt} = 0.0304 = 3.04\%$

$2008: \frac{1}{P} \frac{dP}{dt} = 0.0245 = 2.45\%$

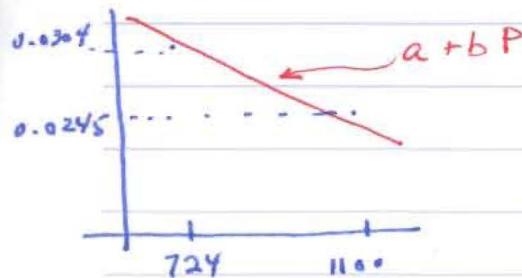
c) To answer this use

$$\frac{1}{P} \frac{dP}{dt} = a + bP$$

since you are told there is a linear relation between $\frac{1}{P} \frac{dP}{dt}$ and P

Use the following 1993: $P = 724$, $\frac{1}{P} \frac{dP}{dt} = 0.0304$

2008: $P = 1100$, $\frac{1}{P} \frac{dP}{dt} = 0.0245$



$$\text{Slope} = \frac{0.0245 - 0.0304}{1100 - 724} = -1.569 \times 10^{-5}$$

intcept by plugging in 1 point in

$$0.0304 = a + (-1.569 \times 10^{-5})(724)$$

$$\therefore a = 0.0418$$

d) Go back to logistic equation in form $\frac{1}{P} \frac{dP}{dt} = k - \frac{k}{L} P$

$$\therefore k = 0.0418 \quad \therefore \frac{k}{L} = 1.569 \times 10^{-5}$$

L = maximum oil
(carrying capacity)

$$\therefore L = \frac{k}{1.569 \times 10^{-5}} = 2662 \text{ bn barrels}$$

(2)

e) The DE becomes $\frac{1}{P} \frac{dP}{dt} = k(1 - \frac{P}{L})$ which can be

separated as $\frac{dP}{P(1-P/L)} = k dt$ and integrated to give

$$P = \frac{P_0 L}{P_0 + (L - P_0)e^{-kt}}$$

where $L = 2662$, P_0 = oil at a known date
 $k = 0.0418$ = choose 1993

$$P_0 = 724$$

$$\therefore \frac{L - P_0}{P_0} = 2.667$$

$$P(t) = \frac{2662}{1 + 2.667 e^{-0.0418 t}}$$