

Math 205 Assignment #9 Solutions

1)

#83 pg 485

$$P(r) = 400 e^{-0.2r^2}$$


 $2\pi r dr$ = area at distance r

$$\therefore d\text{Amount} = (1000)(2\pi r dr)(400 e^{-0.2r^2})$$

$$\therefore \text{Amount (value of hours)} = 8\pi \times 10^5 \int_0^7 r e^{-0.2r^2} dr = -8\pi \times 10^5 [2.5e^{-0.2r^2}]_0^7$$

= 6.28 billion dollars

$$\#85 \text{ mileage (efficiency)} f = 25 + 0.1t, \text{ speed } v = 50 \frac{t}{t+1}$$

$$\text{Fuel consumed} = \int_2^3 \frac{v}{f} dt = \int_2^3 \frac{50 \frac{t}{t+1}}{25 + 0.1t} dt, \text{ let } v = \frac{50t}{t+1}$$

$$\therefore \text{Fuel consumed} = \int_2^3 \frac{10t}{6t+5} dt = 1.246 = 1.25 \text{ gallons}$$

* (note: students can also find distance and divide by average fuel consumption)

2) #24 pg 607

$$\frac{dy}{dt} = y^2(1+t); y=2, t=1$$

$$\therefore \int \frac{dy}{y^2} = \int (1+t) dt \Rightarrow -\frac{1}{y} = t + \frac{1}{2}t^2 + C$$

$$\therefore y^2 = -\frac{1}{t + \frac{1}{2}t^2 + C}$$

$$y(1)=2 \quad \therefore y = -\frac{1}{1 + \frac{1}{2} + C}$$

$$C = -7/4$$

$$\therefore y^2 = -\frac{1}{t + \frac{1}{2}t^2 - 7/4}$$

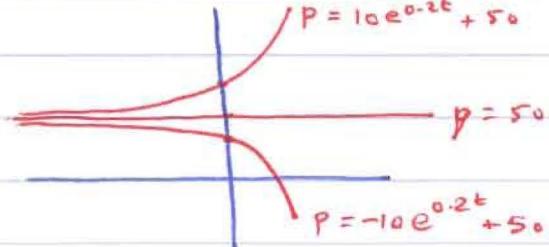
$$y = \sqrt{-\frac{1}{t + \frac{1}{2}t^2 - 7/4}}$$

NDSU

$$\#30) \frac{dP}{dt} = 0.2P - 10 = 0.2(P-50) \quad \therefore \int \frac{dP}{P-50} = \int 0.2 dt$$

$$\therefore \ln(P-50) = 0.2t + C \quad \therefore P-50 = Ce^{0.2t}$$

$$\therefore P = Ce^{0.2t} + 50$$



$$P(0) = 40 \Rightarrow C = -10$$

$$P(0) = 50 \Rightarrow C = 0$$

$$P(0) = 60 \Rightarrow C = 10$$

3) #26 pg 627

There are lots of ways to do this - Basic idea is that the rate of change of the concentration of CO is $\frac{\text{amount in} - \text{amount out}}{\text{volume}}$

It makes sense that eventually the air in the room is 5% CO but it may take some time for this so ...

let $A = \text{amount CO entering in cubic meters} = (5\%)(0.002 \text{ m}^3/\text{min})$

$d = \text{exchange rate} = 0.002 \text{ m}^3/\text{min}$

$C = \text{concentration}$

$$\therefore \frac{dc}{dt} = \frac{A - dc}{V} \quad \therefore \frac{dc}{A - dc} = \frac{1}{V} dt$$

$$\therefore -\ln(A - dc) = \frac{d}{V} dt \quad \therefore A - dc = C_0 e^{-\frac{d}{V} t}$$

$$b) \therefore C(t) = \frac{1}{d} (A - C_0 e^{-\frac{d}{V} t})$$

$$c) \lim_{t \rightarrow \infty} C(t) = \frac{A}{d} = \frac{5\%}{d} \quad \therefore \text{in the long run the room has the same concentration as the outside air from smokers!}$$

3) #8 pg 635

a) you have 3 possible solutions:

$$b) \frac{dP}{dt} = 3P(1-P^2) \quad \therefore \frac{dP}{dt} = 0, P = 1$$

stable (steady) pop at $P = 1$

- c) if $P > 1$ then the rate γ change \rightarrow negative and pop must decrease, etc
 d) $\frac{dP}{dt} > 0$ growth, $\frac{dP}{dt} = 0$ stable, $\frac{dP}{dt} < 0$ decline

#19 $\frac{1}{P} \frac{dP}{dt} = 0.3 \left(1 - \frac{P}{100}\right)$ $P_i = 75$

General solution $P(t) = \frac{L}{1 + Ae^{-\lambda t}}$, $A = \frac{L - P_0}{P_0}$ $\therefore A = \frac{100 - 75}{75}$
 $= 1/3$

$$P(t) = \frac{100}{1 + \frac{1}{3}e^{-0.3t}} \quad \lambda = 0.3$$

$$\frac{dP}{dt} \text{ max at } t = \frac{1}{\lambda} \ln A = \frac{1}{0.3} \ln(1/3) = -\frac{\ln(3)}{0.3}$$

