Math 205 MT#1 Solutions

① a) $\int \ln(e^{-x^2})dx = \int -x^2 dx = -\frac{x^3}{3} + c$

b) $\int \frac{\sin^2 t}{1-\cos^2 t}\, dt = \int dt = t + c$

② a) $\int \frac{\sin t}{1+\cos t}\, dt$ let $u = 1+\cos t$, $du = -dt \sin t$ $\Rightarrow \int \frac{1}{u}\, du = -\ln(1+\cos t) + c$

b) $\int \frac{1}{x\ln(x)}\, dx$ $u = \ln c$, $du = 1/x\, dx$ $= \int \frac{1}{u}\, du = \ln(u) = \ln(\ln(x)) + c$

③ a) $\int \cos(\sqrt{x})dx$ $u = \sqrt{x}$ $\therefore u^2 = x$, $2u\,du = dx$ $= 2\int u \cos u\, du$

$u = u$ $v = \sin u$
$u' = 1$ $v' = \cos u$ $\therefore$ $2\int u\cos u\, du = 2\left(u \sin u - \int \sin u\, du\right)$

$= 2u\sin u + 2\cos u + c$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + c$

④ a) $\int \sin(\ln x)\, dx$ $u = \ln x$ $\therefore x = e^u$, $du = \frac{1}{x}dx$ $\therefore dx = e^u du$

$= \int e^u \sin u\, du$ $\therefore$ use integration by parts $u = e^u$ $v = -\cos u$, $u' = e^u$ $v' = \sin u$

$\int e^u \sin(u)\, du = -e^u \cos u + \int e^u \cos u\, du$ do it again $u = e^u$ $v = \sin u$, $u' = e^u$ $v' = \cos u$

$\int e^u \sin(u)\, du = -e^u \cos u + e^u \sin u - \int e^u \sin u\, du$

$\therefore 2\int e^u \sin u\, du = e^u(\sin u - \cos u)$ $\therefore \int e^u \sin u\, du = \frac{1}{2}e^u(\sin u - \cos$

$= \frac{1}{2}x(\sin(\ln x) - \cos(\ln x)) + c$

b) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ $\quad u = \sin^{-1}x \quad du = \frac{1}{\sqrt{1-x^2}}\,dx \quad \Rightarrow \int u\,du = \frac{u^2}{2} + C$

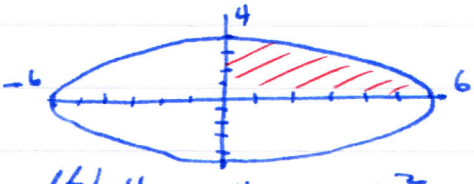
$= \frac{(\sin^{-1}x)^2}{2} + c$

c) $\int (\ln x)^2\,dx$ use parts $\quad u = (\ln x)^2 \quad v = x$

$u' = \frac{2\ln x}{x} \quad v' = 1$

$= x(\ln x)^2 - 2\int \ln x\,dx = x(\ln x)^2 - 2x\ln x + 2x + C$

5 a) $\frac{x^2}{36} + \frac{y^2}{16} = 1$



b) Easiest to find 1/4 and mult by 4 $\quad \frac{y^2}{16} = 1 - \frac{x^2}{36}$

$\therefore y^2 = 16 - \frac{16x^2}{36}$, $y = 4\sqrt{1-\left(\frac{x}{6}\right)^2}$ so, the area 2 $\frac{1}{4}$ of the pizza $= \int_0^6 4\sqrt{1-\left(\frac{x}{6}\right)^2}\,dx = 4\int_0^6 \sqrt{1-\left(\frac{x}{6}\right)^2}\,dx$; let $u = \frac{x}{6}$ $\therefore 6\,du = dx$

c) $= 24\int_0^u \sqrt{1-u^2}\,du = 24\left(\frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\sin^{-1}u\right) = 24\left(\frac{x}{12}\sqrt{1-\frac{x^2}{36}} + \frac{1}{2}\sin^{-1}\frac{x}{6}\right)\Big|_0^6$

$= \frac{24\pi}{4}$

All 4 parts $= 4 \times \left(\frac{24\pi}{4}\right) = \underline{24\pi}$

$\therefore$ Area of pizza is 24π in^2 price is \$ $\underline{2.4\pi}$ = $\underline{\$7.54}$