

Math 205

Mid term #2

Solutions

1) a) $\frac{dz}{dt} = z + ze^t = z(1+e^t) \quad \therefore \frac{dz}{z} = (1+e^t)dt$

$$\int \frac{dz}{z} = \int (1+e^t)dt \quad \therefore \ln z = t + \frac{1}{2}e^t + C \quad \therefore z = Ce^{t + \frac{1}{2}e^t}$$

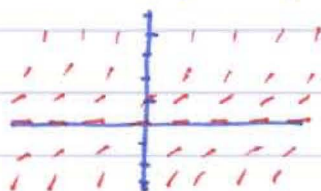
$z=5, t=0 \quad \therefore C=5$ so $z(t) = 5e^{t + \frac{1}{2}e^t}$

b) $\frac{dy}{dx} = \frac{4x}{y^2} \Rightarrow y^2 dy = 4x dx \quad \therefore \int y^2 dy = \int 4x dx$

$$\therefore \frac{1}{3}y^3 = 2x^2 + C \quad \therefore y = \sqrt[3]{6x^2 + C}$$

$y(0)=1 \quad \therefore C=1$ so $y(x) = \sqrt[3]{6x^2 + 1}$

c) $\frac{dy}{dx} = y^2$



2) a) The population change is logistic - this can be seen by inspecting the equation. This means that the pop will either grow to a stable value or decline to a stable value - only choice B shows this.

b) $\frac{dP}{dt} = 0.05P - 0.0005P^2 = 0.05P(1 - 0.01P) = 0.05P(1 - \frac{P}{100})$

$\therefore k = 0.05$ and $L = 100$, so the carrying capacity for the lake is 100 (thousand) fish of this species.

c) Solve $\frac{dP}{dt} = kP(1 - \frac{P}{L})$ by separating and using partials

$$\therefore \frac{dP}{P(1 - \frac{P}{L})} = k dt \quad \frac{1}{P(1 - \frac{P}{L})} = \frac{1}{P} + \frac{1/L}{1 - P/L}$$

$$\therefore \int (\frac{1}{P} + \frac{1/L}{1 - P/L}) dP = \int k dt = kt \quad \therefore \ln P - \ln(1 - P/L) = kt$$

$$\therefore \ln(\frac{1 - P/L}{P}) = -kt + c \quad \therefore \frac{1 - P/L}{P} = Ce^{-kt} \quad \text{At } t=0 \text{ to get } c$$

$$\therefore \frac{1 - P_0/L}{P_0} = c \quad \therefore C = \frac{L - P_0}{P_0} \quad \text{or} \quad \frac{100 - P_0}{P_0} = A \text{ which is the name I gave it}$$

$$\therefore P(t) = \frac{100}{1 + Ae^{-kt}} \quad k = 0.05$$

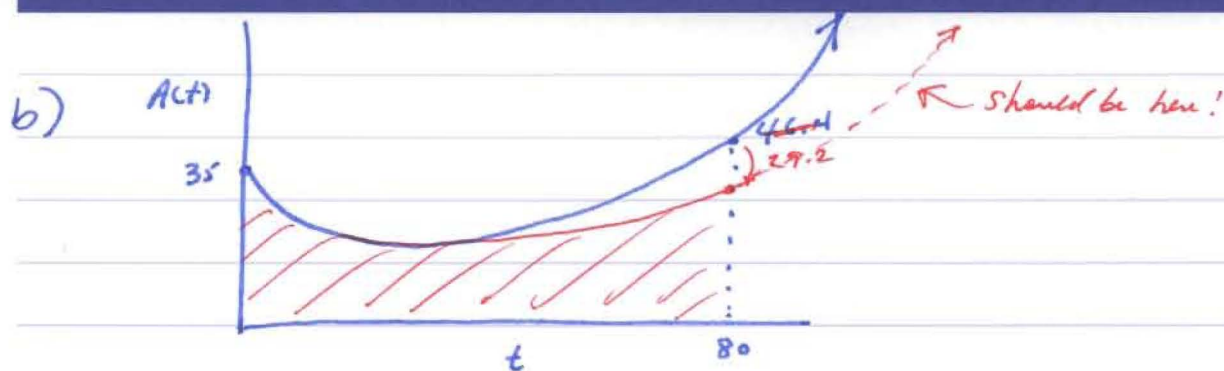
d) you have already exceeded the carrying capacity so the first population will "relax" back to L

$$\lim_{t \rightarrow \infty} P(t) = \frac{100}{1 + A(e^{-0.05t})} \text{ goes to zero} \quad \therefore P \rightarrow 100$$

3 a) 1 function is steadily dropping ($\frac{50}{2 + 0.3t}$) while the other is steadily growing ($10e^{-0.0125t}$).

When $t=0$ the first function = 25, the second = 10
 so $A(0) = 35$, when $t=80$ $A(80) = \frac{50}{2 + 0.3(80)} + 10e^{0.0125 \cdot 80}$

$$\therefore A(80) = 19.2 + 29.2 = 48.4 \quad \text{If you pick a value between } t=0 \text{ and } t=80 \text{ you will get a lower value than either so it is reasonable that the function has a minimum}$$



c) The student expended the most brainwave energy at ~~$t=80$~~ $t=0$

d) The total brainwave energy emitted = $\int_0^{80} A(t) dt$

$$= \int_0^{80} \left(\frac{50}{2+0.3t} + 10e^{0.0125t} \right) dt$$

$$= \frac{50}{0.3} \ln(2+0.3t) \Big|_0^{80} + 800 e^{0.0125t} \Big|_0^{80} = 167 \ln(2+0.3t) \Big|_0^{80} + 800 e^{0.0125t} \Big|_0^{80}$$

$$= 428.3 + 1374.6 = \underline{\underline{1802.9 \text{ mJ}}}$$

e) The average rate of brain-wave activity is just

$$\frac{\int_0^{80} A(t) dt}{80 - 0} = 22.5 \text{ mJ/min}$$