

Math 205 - MidTerm # 1 B

$$1) \quad a) \int \frac{e^x}{\sqrt{\sin^2(x^3) + \cos^2(x^3)}} dx = \int \frac{e^x}{\sqrt{1}} dx = e^x + C$$

$$b) \int \underbrace{\sin(t+1)\cos(t+1)}_{\rightarrow} dt = \frac{1}{2} \int \sin 2(t+1) dt = -\frac{1}{4} \cos 2(t+1) + C$$

$$c) \int \frac{t}{\sin^{-1}(\sin t^2)} dt = \int \frac{t}{t^2} dt = \int \frac{1}{t} dt = \ln|t| + C$$

$$2) \text{ a) } \int \frac{\sec^2 x}{\tan x} dx \quad \text{let } u = \tan x \quad \frac{du}{dx} = \sec^2 x \quad \therefore = \int \frac{1}{u} du = \ln|u| + C = \ln|\tan x| + C$$

$$b) \int \frac{1}{\sqrt{x^2 - 4x - 5}} dx = \int \frac{1}{\sqrt{x^2 - 4x + 4 - 9}} dx = \int \frac{1}{\sqrt{(x-2)^2 - 9}} dx \quad \text{let } u = x-2 = \int \frac{1}{\sqrt{u^2 - 1}} du$$

use identity #43 $\therefore = \ln |(x-2) + \sqrt{(x-2)^2 - 9}| + C$

$$c) \int 30x e^{5x^2} dx \quad \text{Let } u = 5x^2 \quad \therefore \int 3e^u du = 3e^u + C = 3e^{5x^2} + C$$

$$3) \quad \int x^2 e^x dx \quad f = x^2 \quad g = e^x \quad \Rightarrow \quad x^2 e^x - \int 2x e^x dx \quad f = x \quad g = e^x \\ f' = 2x \quad g' = e^x \quad f' = 1 \quad g' = e^x \\ \therefore \quad x^2 e^x - 2 \left(x e^x - \int e^x dx \right) = x^2 e^x - 2x e^x + 2 e^x + C$$

$$4) \quad a) \quad \int \frac{x}{\sqrt{3-2x-x^2}} dx = \int \frac{x}{\sqrt{2+1-2x-x^2}} dx = \int \frac{x}{\sqrt{2+(1-x)^2}} dx \quad \text{let } \begin{cases} u = 1-x \\ du = -dx \end{cases} \quad \therefore x = 1-u$$

$$= - \int \frac{1-\mu}{\sqrt{2+\mu^2}} d\mu = \int \frac{\mu}{\sqrt{2+\mu^2}} d\mu - \int \frac{1}{\sqrt{2+\mu^2}} d\mu \quad \text{use tables! for 2nd integral}$$

$$= \sqrt{2+u^2} + \sinh^{-1}\left(\frac{\sqrt{2}}{2}u\right) = \sqrt{2+(1-x)^2} + \sinh^{-1}\left(\frac{\sqrt{2}}{2}(1-x)\right) + C$$

$$b) \int \frac{t}{\sqrt{1-t^2}} dt \quad \sin^2 \mu + \cos^2 \mu = 1 \quad \therefore \sin(\cos^{-1} t) = \sqrt{1 - (\cos(\cos^{-1} t))^2} = \sqrt{1 - t^2}$$

$$\therefore \int \frac{t}{\sqrt{1-t^2}} dt = -\sqrt{1-t^2} + C$$

$$c) \int_0^{\pi/2} \frac{\cos t}{\sqrt{1 + \sin^2 t}} dt \quad \text{let } u = \sin t \quad du = \cos t dt \quad \Rightarrow \quad \int_0^{\pi/2} \frac{1}{\sqrt{1 + u^2}} du ; \text{ use tables} = \sinh^{-1}(u) \Big|_{0}^{\pi/2} \\ = \sinh^{-1}(\sin x) \Big|_0^{\pi/2} \\ = \sinh^{-1}(1) - \sinh^{-1}(0) \\ = \sinh^{-1}(1)$$