

## SPECIAL FUNCTIONS

## EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$$\log_a x = y \iff a^y = x$$

$$\ln x = \log_e x, \text{ where } \ln e = 1$$

$$\ln x = y \iff e^y = x$$

## Cancellation Equations

$$\log_a(a^x) = x \quad a^{\log_a x} = x$$

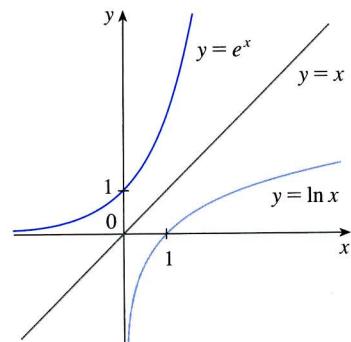
$$\ln(e^x) = x \quad e^{\ln x} = x$$

## Laws of Logarithms

$$1. \log_a(xy) = \log_a x + \log_a y$$

$$2. \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$3. \log_a(x^r) = r \log_a x$$

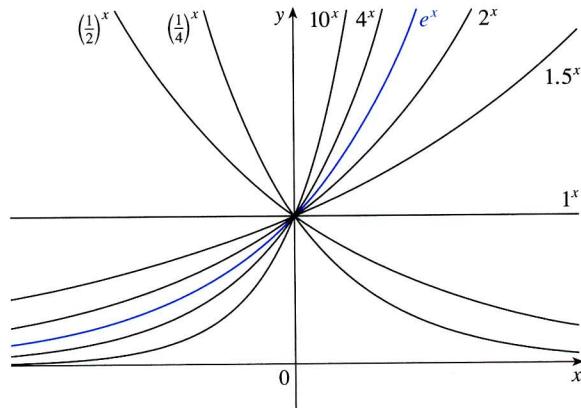


$$\lim_{x \rightarrow -\infty} e^x = 0$$

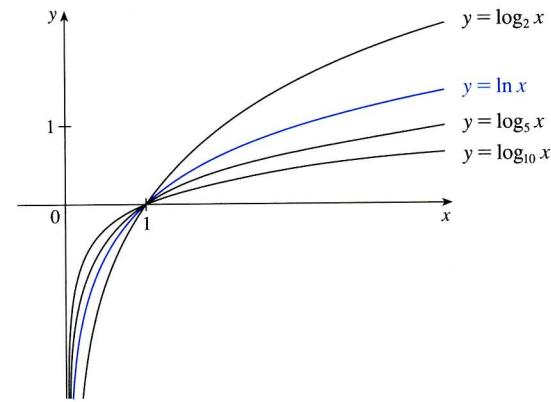
$$\lim_{x \rightarrow \infty} e^x = \infty$$

$$\lim_{x \rightarrow 0^+} \ln x = -\infty$$

$$\lim_{x \rightarrow \infty} \ln x = \infty$$



Exponential functions



Logarithmic functions

## HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

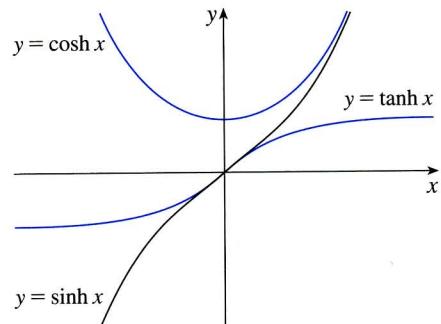
$$\csc x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$



## INVERSE HYPERBOLIC FUNCTIONS

$$y = \sinh^{-1} x \iff \sinh y = x$$

$$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$$

$$y = \cosh^{-1} x \iff \cosh y = x \text{ and } y \geq 0$$

$$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$$

$$y = \tanh^{-1} x \iff \tanh y = x$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$