Copper Pipe Xylophone

EQUIPMENT

- ¾" Copper pipes
- Different diameter pipes with same lengths
- Mallets
- Weather-strip coated board stands for the copper pipes
- Tuners
- Rulers or tape measures
- Microphones, stands, preamps connected to computers running Adobe Audition.
- Pipe cutters
- Stand to hold prepared (tuned) lengths of pipe
- Excel spreadsheet xylophone.xlsx

INTRODUCTION

For a guitar string or a column of air, the pitch of the fundamental tone sounded is proportional to the length of the string or length of the column of air. However for other systems the pitch of the fundamental tone may depend on the length in a more complicated manner. For example the pitch of the fundament al model may depend on the square of the length or the square root of the length. In this lab we will experimentally measure the way that the fundamental tone of a bending copper pipe depends on its length. We can write

$$f \propto L^{\alpha}$$

where f is the frequency of the fundamental tone, L is the length of the pipe and α is a power that we can measure. The symbol $\propto$ means "is proportional to". For guitar strings and flutes, $\alpha \approx -1$, and the pitch of the fundamental tone is inversely proportional to the length of the string or column of air.

If we take the log of the above equation we find

$$\log f = \alpha \log L + \text{constant}$$

On a plot of $\log f$ vs $\log L$ the exponent α would be the slope of a line.

For a guitar string or a column of air, the overtones are integer multiples of the fundamental tone. However there are vibrating systems where the overtones are not integer multiples. This contributes to their timbre. Bells, drums and copper pipes are examples of instruments that have a complex spectrum of overtones. In this lab you will measure the frequencies of these overtones, f_n and their ratios to that of the fundamental or f_n/f_1. Here f_n refers to the n-th partial or overtone. As explored in the book by Hopkins the ratios of the frequencies depends on the shape of the vibrating object. By shaving off material from regions of a metal or wood bar, the ratios of the frequencies can be varied.

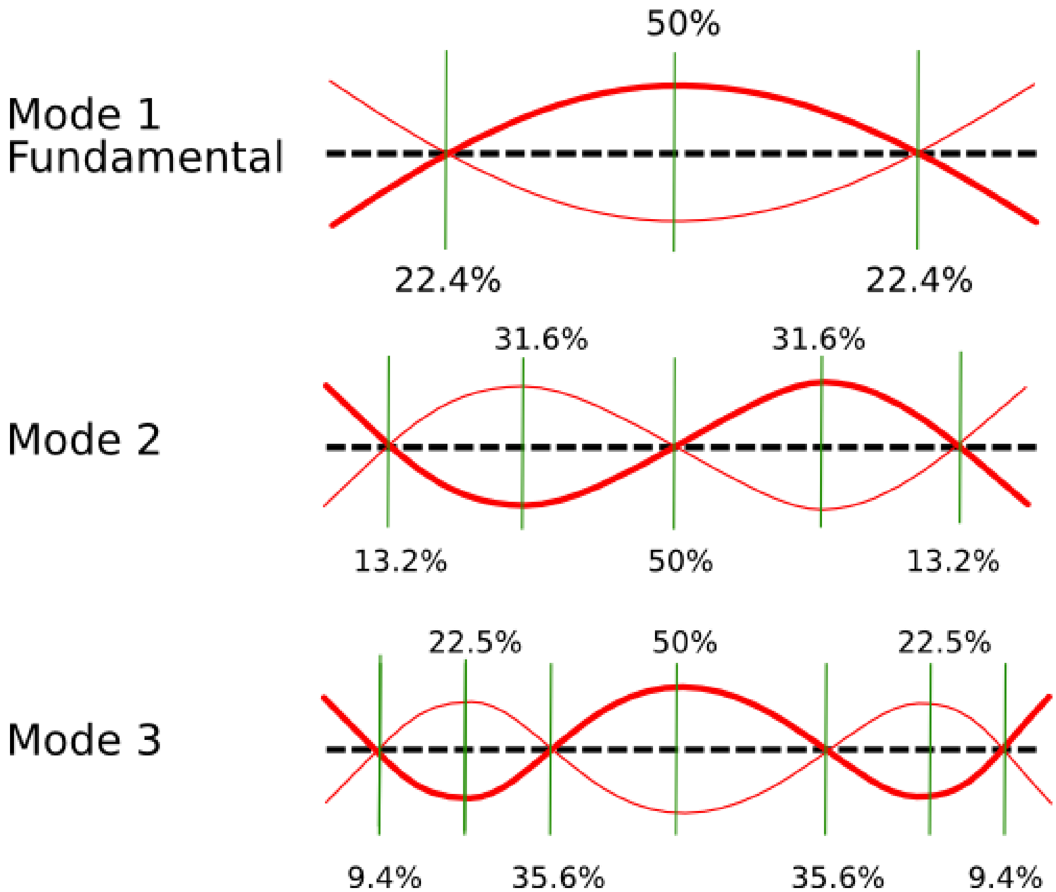

Figure 1. This figure shows motions for first three modes of a steel bar. The steel bar is not help fixed at either end. Based on a Figure by Bart Hopkins.

The frequencies of the modes excited in a copper pipe depend on the speed of sound in the pipe and the stiffness of the pipe. A different diameter pipe should have a different stiffness (harder or easier to bend) and so should have different frequencies of vibration. A copper pipe has bending modes similar to those in a steel bar shown above.

By measuring the frequencies of the fundamental bending mode for copper pipes of different lengths we can determine experimentally how the fundamental model frequency depends on pipe length. Specifically we can measure the exponent α in Equation 1 or 2 above.

When a copper pipe is hit it moves with bending modes (shown above) that have frequency that depend on pipe length. We would expect that all three modes shown above would have higher frequencies when the pipe is shorter. However the pipe can also deform in other ways. For example two sides of the pipe could approach each other while the opposite sides move away (see Figure 2) . The frequencies of these modes would not depend on pipe length, though they would be sensitive to pipe thickness and diameter. In this lab we can look at the spectrum of a copper pipe to see if we can find mode frequencies that don't depend on pipe length.

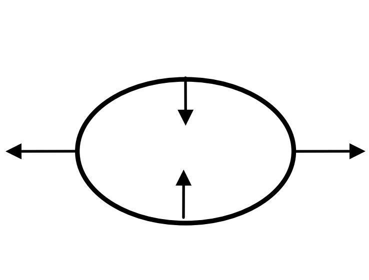

Figure 2. Possible vibration modes looking down the end of a copper pipe. This type of motion could correspond to a mode with frequency that does not depend on pipe length.

If a slot is cut in the end of the pipe then the ends of the pipe an also move away from each other, in a way similar to a tuning fork (see Figure 3).

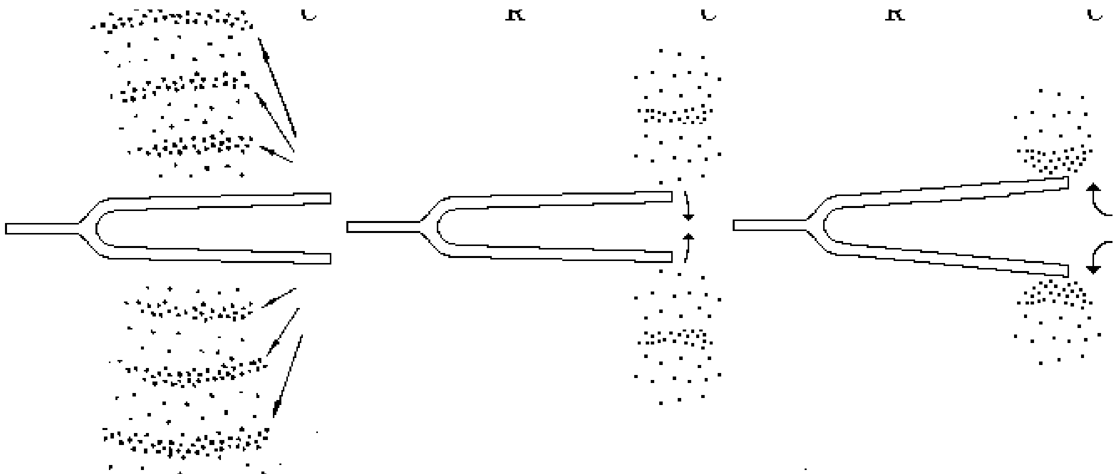

Figure 3. This figure shows motions for a tuning fork. We expect a lower fundamental mode frequency if the fork prongs are longer.

A copper pipe with a slit cut in the end has many possible modes of vibration leading to a rich spectrum and possibly a pleasing sound. In this lab we will look at how the spectrum of a copper pipe changes as a slit is cut into its end.

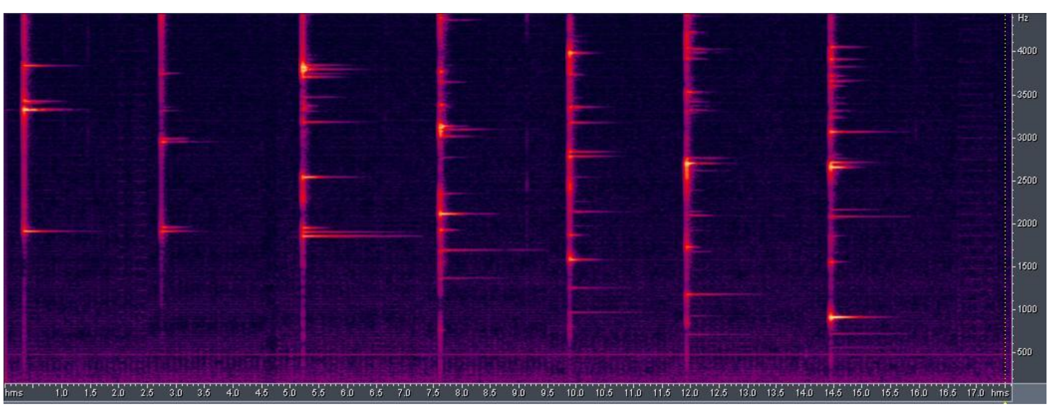

Figure 4. Spectra of a ¾" diameter, 9" long copper pipe. On the left no slit has been cut. From the left to the right each spectrum corresponds to the pipe with a 1cm longer slit. Low frequency modes are seen when the slit is large enough that slow tuning fork modes are possible. The last spectrum with the 6cm slit had a nice sound. Perhaps two modes

coincide and the lowest frequency mode is particularly strong as a consequence. Because I liked the sound I stopped extending the slit.

PROCEDURE

A. Pitch as a function of length.

1. Prepare 3/4" pipes at different lengths set up as a xylophone playing a scale. Hit the pipes and measure the frequencies of their lowest tones.
2. What notes are played?
3. What is the relation between pipe length and pitch? Plot pipe length vs pitch for the 12 pipes. Plot log pipe length vs log pitch for the 12 pipes. On which of these plots do the points lie on a line?
4. Does the frequency depend linearly on the length of the pipe? What is your best estimate for α in equation 1? The line that best goes through your data points should determine your best estimate (measurement) for α.

B. Pitch as a function of pipe diameter and material

1. Measure the fundamental or lowest frequency of vibration for two pipes of the same material (copper) but different diameters.
2. Suggest a relationship between pipe diameter and fundamental frequency.
3. Measure the fundamental frequency for two pipes of the same length and approximately same diameter but different material (steel and copper). Are the frequencies higher or lower for a stronger material?

C. Structure of Overtones

1. The sound of the pipe may depend upon where you hold the pipe or where you hit it. Record while you hit the pipe in different locations or hold the pipe at different locations. Does the frequency spectrum change?
2. Measure the overtones for two different length 3/4" pipes. Compute the ratios of the overtones divided by the fundamental. Are these ratios the same for the two different diameter pipes that have the same length? Note that these ratios are not integers (with rare exceptions).
3. Record while you play a scale on the copper pipe lengths. Make a figure similar to Figure 4. You can cut and paste audio to make it look nice. Take a snap shot of your Figure for your lab report! (Command shift 4 and then mail it to yourself!) Can you see any patterns in the overtone spectra. Which overtones depend on pipe length? Bending modes such as shown in Figure 1 are likely to depend on pipe length, whereas motions like those shown in Figure 2 will not depend on pipe length. You may need to look at overtones up to 5000 Hz or so to find some that don't vary with pipe length. Do all overtones shift by the same amount or do groups of them shift together?
4. Record the three pipes of the same length but different diameter or material. Do all overtones shift by the same amount or do different groups of them shift together? Can you see similarities between groups of overtones? Can you find any patterns in groups of overtone frequencies that tell you about the kinds of motions for these modes?

DISCUSSION

1. For the string and wind instruments frequency is proportional to length. Did you find that fundamental frequency was proportional to pipe length? Why might you see a non-linear relationship between fundamental frequency and length?
2. How well were you able to measure α?
3. Discuss the relation between the fundamental pitch and pipe length, the fundamental pitch and pipe diameter, thickness and the fundamental pitch and pipe material. We might expect stiffer pipes to have higher frequency fundamentals.

4. If you do part C you will have a figure like that shown in Figure 4 showing different overtones for different length pipes (and possibly different diameter and material pipes

LAB REPORTS

- Name, abstract, collaborators.
- Abstract summarizing what you found.
- A measurement for the exponent α based on your measurements. Note if you are confused by logs, assume that α is an integer or a half integer and see which half integer or integer fits the best. For example, you can try α =-1/2, -1, -3/2, -2, and -2.5 and see which exponent might give the best explanation for how the frequency of the copper pipe xylophone scales with length. Note I have only chosen negative exponents because I expect that the frequency is higher for shorter pipes.
- A spectrogram showing a series of spectra (like shown in Figure 4) based on part C or part D or part E.
- A valiant attempt to explain the very complicated set of spectra shown in your spectrogram figure in terms of different types of vibrations.