The King's University College

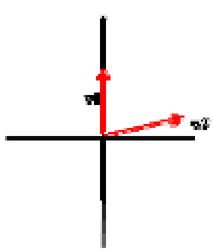
Physics 241 Midterm I (Unit C) - Solution

November 3, 2006

Instructions: You have a maximum of 90 minutes to complete the following 4 questions. Each question is worth a total of 10 marks and is composed of several parts. Please show your work completely. No marks will be awarded if answers are not supported by your work. Please answer in the booklet provided.

1. Two-minute questions. You may simply answer True or False or provide a numerical answer for the following 5, two-minute questions taken from Moore:
   a. In any collision in which momentum is conserved, kinetic energy must also be conserved. (T or F) False
   b. The thermal energy of a block of ice at 0^oC melting into a puddle of water at 0^oC; increases, decreases or stays the same - which one? Since the temperature does not change the thermal energy remains the same.
   c. What is the impulse delivered by a force of 200 N that acts for 1 minute? Use $J = F\Delta t$ to get J = (200 N)(60 s) = 12 000 Ns.
   d. What is the rotational energy of a 10 kg sphere of radius 10 cm spinning at 10 revs/s? ($I_{sphere} = {}^2/_5 Mr^2$) Use $E = 1/2 I\omega^2$ to get $E = 1/2\ [({}^2/_5)(10\ kg)(0.1)^2](20\pi\ 1/s)^2 = 79\ J$
   e. A cup of tea becomes cooler with time. Is this change in thermal energy due to a flow of heat or to work? Heat!

2. A 70 kg skater moving with velocity $\vec{v}_1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix} m/s$ collides with a 40 kg skater moving with velocity $\vec{v}_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} m/s$. The hold on to each other after the collision.
   a. Draw each of these vectors on an x-y grid and explain in words how you would interpret the velocities. (2)



V1 shows an object going only in the y direction at 3 m/s while v2 depicts an object moving both in the x and in the y directions with speeds of 3 m/s and 1 m/s respectively.

   b. Show, by treating this as an isolated system, how Conservation of Momentum can be used to predict the final direction of motion of the skaters and find the final velocity(3) Use $\vec{p}_i = \vec{p}_f$

$$70kg\begin{bmatrix}0\\3\end{bmatrix}m/s + 40kg\begin{bmatrix}3\\1\end{bmatrix}m/s = \begin{bmatrix}120\\250\end{bmatrix}Ns$$

Since the combined mass is 110 kg, you can conclude that

$$\vec{p}_f = 110kg\begin{bmatrix}v_x\\v_y\end{bmatrix} = \begin{bmatrix}120\\250\end{bmatrix}Ns$$

$$v_x = 120/110 = 1.09\ m/s$$
$$v_y = 250/110 = 2.27\ m/s$$
$$\vec{v}_f = \begin{bmatrix}1.09\\2.27\end{bmatrix}m/s$$

c. What is the final speed of the skaters?(2) $v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(1.09)^2 + (2.27)^2} = 2.52\ m/s$

d. Is kinetic energy conserved as well? Explain your answer.(3) No! This is an inelastic collision which means that the maximum possible amount of kinetic energy was lost. To verify this just compare the initial and final kinetic energies: E_i = 515 J while E_f = 349 J.

3. Canadian Olympic medalist Alexandre Despatie begins his dive with his hands together, stretched above his head and rotating with a very small rotational velocity ωi. He then goes into a tight tuck position as shown on the right.

a. Why can this situation be treated as a "floats in space system?" (2 marks)The system is essentially free from any important external forces or torques that would change the rotational momentum of the diver. This is because air resistance is negligible over the brief time of interaction and the gravitational force is essentially constant.

b. Explain what shapes you would use to model the rotational effects both before he assumes the tuck position and after. (2 marks)Model the diver as a rod (stretched position) and a sphere (tuck position).

c. Make reasonable estimates for the dimensions of the diver (height, mass, etc) in the stretched and tuck positions. (2 marks) M = 75 kg, L = 2.75 m and r = 0.75 m

d. Assume the original rotational velocity was 0.5 rads/s. Calculate the final rotational speed of the diver. By what factor has his rotational energy increased? (3 marks) Use conservation of angular momentum:

$$\vec{L}_i = \vec{L}_f$$
$$I_i\vec{\omega}_i = I_f\vec{\omega}_f$$

$I_i = \frac{1}{12}ML^2,\ I_f = \frac{2}{5}MR^2$

solving for ω_f you get: $\omega_f = \frac{I_i}{I_f}\omega_i = \frac{\frac{1}{12}ML^2}{\frac{2}{5}MR^2} = \frac{5}{24}\left(\frac{L}{R}\right)^2\omega_i$

insert your numbers to get something similar to ω_f = 1.4 radians/s. To compute the increase in rotational energy ratio use: $\frac{E_f}{E_i} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{\frac{2}{5} M R^2 \omega_f^2}{\frac{1}{12} M L^2 \omega_i^2} = \left(\frac{24}{5}\right)\left(\frac{R}{L}\right)^2\left(\frac{\omega_f}{\omega_i}\right)^2$

= 2.8

e. Where did the additional energy come from? (1 mark) The additional energy came from work done by the diver as he pulled his legs and arms in tight to his body.

4. A cylindrical drum of mass M = 100 kg and radius r = 0.5 m is released from rest on the track shown at the right.
   a. Discuss all relevant energy transfers that take place as M drops through points A and B and show how conservation of energy can be applied here. (4 marks) Use ΔE=0 to conclude that the change in potential energy plus changes in kinetic and rotational energy must all sum to 0. You can express this via:

      $\Delta Ep + \Delta Ek + \Delta Erot = 0$

      $mg\Delta h + \frac{1}{2}m(v_f^2 - v_i^2) + \frac{1}{2}I(\omega_f^2 - \omega_i^2) = 0$

      als0, assume that there is no slipping ($v = \omega r$) and simplify to get $mg\Delta h + \frac{3}{4}mv^2 = 0$

   b. At what point along the track does the mass experience maximum acceleration? (1 mark) Maximum acceleration will occur at the very top of the track.
   c. Calculate the speed of the drum at points A and B (4 marks). Just insert numbers using Δh = -5 m at A and Δh = -4 m at B. Thus: v_A = 8.1 m/s and v_B = 7.2 m/s.
   d. If the mass where a solid sphere rather than a solid cylinder how would your answers in part c change? Just explain in words - you don't need to calculate this. (1 mark)

      If the mass was spherical in shape then the moment of inertia would be smaller (by 20%) and that would mean that less energy would be converted to a rotational form. Hence the kineti energy and therefore velocity would be a bit higher.