Physics 243 Midterm Solutions

1) a) For SHM to occur $\ddot{x}(t) \propto -Ax(t)$ – the acceleration is negative (opposite) and linearly related to displacement

b) $F = -kx \quad \therefore F = -(25\frac{N}{m})(0.05m) = \underline{\underline{1.25N}}$

c) $x(t) = A\cos\omega t \quad \omega = \sqrt{k/m} \quad \therefore v(t) = -A\omega\sin\omega t$

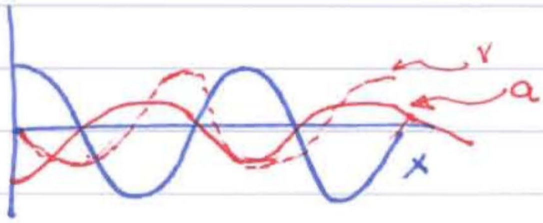
$A = 5cm$

$\frac{1}{2}mv^2(x) + \frac{1}{2}kx^2 = E = \frac{1}{2}kA^2 \quad \therefore \frac{1}{2}mv^2 = \frac{1}{2}k(A^2 - x^2)$

$\therefore v^2 = \frac{k}{m}(A^2 - x^2) \quad \therefore v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \qquad A = 0.05m, \ x = 0.02m$

$\therefore v = \pm\sqrt{\frac{25N/m}{0.2kg}(0.05^2 - 0.02^2)} = 0.51 m/s$

d)



2) a) Wave: diffraction, interference, polarization, E-M theory
Particle: photoelectric effect, compton scattering

b) $E = hf - W_0 \quad \therefore W_0 = hf - E \qquad E = 0.5eV = 0.8\times10^{-19}J$

$\therefore W_0 = \frac{hc}{\lambda} - E = \frac{(6.63\times10^{-34})(3\times10^{8})}{425\times10^{-9}} - 0.8\times10^{-19}J \qquad h = 6.63\times10^{-34}J\cdot s, \ f = \frac{c}{\lambda}$

$= \underline{\underline{2.4eV}}$

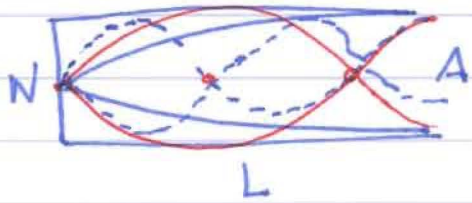
c) Both particles have the same charge and so will have the same energy after accelerating through the same potential

$K = \frac{p^2}{2m}$ $\quad p = \frac{h}{\lambda}$ $\quad \therefore K = \frac{h^2}{2m\lambda^2}$ $\quad \therefore \lambda \propto \frac{1}{\sqrt{m}}$

$\therefore \frac{\lambda_e}{\lambda_p} = \frac{1/\sqrt{m_e}}{1/\sqrt{m_p}}$ $\quad \therefore \frac{\lambda_e}{\lambda_p} = \sqrt{m_p/m_e}$ $\quad \therefore \lambda_e = \sqrt{\frac{m_p}{m_e}}\, \lambda_p$

$\therefore \lambda_e \approx 45 \lambda_p$ Proton has smallest λ

3)
a)



$L = \frac{1}{4}\lambda_1 \quad \therefore \lambda_1 = 4L$

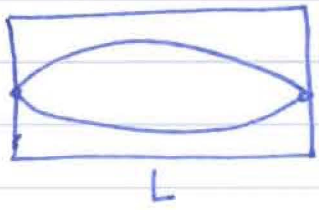
$L = \frac{3}{4}\lambda_3 \quad \therefore \lambda_3 = \frac{4}{3}L$

$L = \frac{5}{4}\lambda_5 \quad \lambda = \frac{4}{5}L$

$\therefore \lambda_n = \frac{4L}{n} \quad n = 1, 3, 5, \cdots$

b) $f = \frac{v}{\lambda}$ $\quad \therefore f_1 = \frac{344\,\text{m/s}}{4(0.1\,\text{m})} = \underline{\underline{860\,\text{Hz}}}$, $\quad f_3 = 3f_1 = \underline{\underline{2580\,\text{Hz}}}$

c)

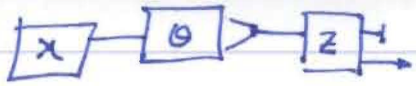


$\lambda = 2L$ the de Broglie wavelength $\lambda = \frac{h}{p}$ is 2 nm This means that the electron has momentum! It can't be at rest.

$\therefore E = \frac{p^2}{2m} = \frac{h^2/\lambda^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{8mL^2} = 6 \times 10^{-20}\,\text{J}$

$= \underline{\underline{0.38\,\text{eV}}}$

4) a)



With both θ channels combined (without observing) the θ machine is effectively removed ∴ you should get 50% out the −z channel

b) To show this consider the sum of the amplitudes for each path:

$$\langle -z|+\theta\rangle\langle+\theta|+x\rangle + \langle -z|-\theta\rangle\langle-\theta|+x\rangle$$

$$= [0,1]\begin{bmatrix}\cos\tfrac{1}{2}\theta\\ \sin\tfrac{1}{2}\theta\end{bmatrix}[\cos\tfrac{1}{2}\theta, \sin\tfrac{1}{2}\theta]\begin{bmatrix}\sqrt{\tfrac{1}{2}}\\ \sqrt{\tfrac{1}{2}}\end{bmatrix} + [0,1]\begin{bmatrix}-\sin\tfrac{1}{2}\theta\\ \cos\tfrac{1}{2}\theta\end{bmatrix}[-\sin\tfrac{1}{2}\theta, \cos\tfrac{1}{2}\theta]\begin{bmatrix}\sqrt{\tfrac{1}{2}}\\ \sqrt{\tfrac{1}{2}}\end{bmatrix}$$

$$= (\sin\tfrac{1}{2}\theta)\left(\frac{1}{\sqrt{2}}(\cos\tfrac{1}{2}\theta + \sin\tfrac{1}{2}\theta)\right) + (\cos\tfrac{1}{2}\theta)\left(\frac{1}{\sqrt{2}}(-\sin\tfrac{1}{2}\theta + \cos\tfrac{1}{2}\theta)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\cancel{\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta} + \sin^2\tfrac{1}{2}\theta - \cancel{\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta} + \cos^2\tfrac{1}{2}\theta\right) = \frac{1}{\sqrt{2}}$$

1

$$\therefore \quad Prob = |amplitude|^2 = \tfrac{1}{2}$$

c) Now block the −θ channel. The path now has an amplitude

$$\langle -z|+\theta\rangle\langle+\theta|+x\rangle = [0,1]\begin{bmatrix}\cos\tfrac{1}{2}\theta\\ \sin\tfrac{1}{2}\theta\end{bmatrix}[\cos\tfrac{1}{2}\theta, \sin\tfrac{1}{2}\theta]\begin{bmatrix}\sqrt{\tfrac{1}{2}}\\ \sqrt{\tfrac{1}{2}}\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}(\sin\tfrac{1}{2}\theta)(\cos\tfrac{1}{2}\theta + \sin\tfrac{1}{2}\theta) \quad \therefore \quad P_r = \frac{1}{2}\sin^2\tfrac{1}{2}\theta\,(\cos^2\tfrac{1}{2}\theta + 2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta + \sin^2\tfrac{1}{2}\theta)$$

1

$$= P_r = \frac{1}{2}\sin^2\tfrac{1}{2}\theta\left(1 + \underbrace{2\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta}_{\sin\theta}\right) = \frac{1}{2}\sin^2\tfrac{1}{2}\theta\,(1+\sin\theta)$$

For $90^\circ < \theta < 180^\circ$ this will be $> \tfrac{1}{2}$ (you can see this if you graph $Pr(\theta)$)