# The Wave Nature of Light 

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## Chapter Overview

## Introduction

In this chapter we examine interference and diffraction of waves moving in two and three dimensions and discuss how the observation of interference and diffraction ef fects in light offers strong evidence for the wavelike nature of light.

## Section Q2.1: Two-Slit Interference

Two-dimensional waves going through a small opening in a barrier (a slit) fan ou into circular waves (whose crests are circles centered on the slit), as if the slit were a point source for the waves. This phenomenon is called diffraction. The superpositior principle implies that circular waves emerging from two closely spaced slits will interfere with each other in the region beyond the slits where the waves overlap, con structively interfering at some points and destructively interfering at others. A straightforward geometric argument implies that if wave crests emerge from both slits simultaneously, we can locate points where the waves interfere constructively by using the following equation:

$$
\begin{equation*}
d \sin \theta_{n c}=n \lambda \quad \Rightarrow \quad \theta_{n c}=\sin ^{-1} \frac{n \lambda}{d} \tag{Q2.1}
\end{equation*}
$$

Purpose: The set of all points at which waves emerging from two slits in a barrier constructively interfere forms lines that radiate from the point halfway between the slits. This equation specifies the angles $\theta_{n c}$ that those lines make with the direction perpendicular to the barrier.

Symbols: $\lambda$ is the wavelength of the waves, $d$ is the center-to-center distance between the slits, and $n$ is an integer.

Limitations: This equation is accurate only at points very distant from the slits compared to $d$. The slits must be arranged so that a given crest of the line wave moves through both slits simultaneously. The slits need to be comparable in width to $\lambda$ to generate circular waves that are strong at all physically reasonable angles (see section Q2.3).

This equation also applies to three-dimensional waves (such as sound waves) if w measure the waves on a plane that is perpendicular to the barrier containing the slit but contains the line connecting the centers of the slits.

## Section Q2.2: Two-Slit Interference of Light

In the first decade of the 1800s, Thomas Young was able to demonstrate that light witl a well-defined color on the rainbow (monochromatic light) emerging from two slit in an opaque barrier created an interference pattern that was accurately described by equation Q2.1. This provided the first strong evidence that light might consist o waves (not particles, as Newton had presumed).

## Section Q2.3: Diffraction

Huygens's principle states that each point in the crest of a wave acts as if it were a point source of circular wavelets, and that the wave's crest at a later time is tangent to the wavelet crests emitted by all points on the crest at the earlier time. Using this principle, we can argue that waves moving through a single slit will create a pattern involving a central range of angles where the waves are strong flanked by fringes of weaker waves, all separated by specific angles where the waves suffer complete destructive interference:

$$
\begin{equation*}
\theta_{n d}=\sin ^{-1} \frac{n \lambda}{a} \tag{Q2.10}
\end{equation*}
$$

Purpose: The points at which waves emerging from a single rectangular slit completely cancel out lie along lines that radiate from the slit's center. This equation specifies the angles $\theta_{n d}$ between those lines and the direction of motion of the original waves moving through the slit.

Symbols: $\lambda$ is the wavelength of the waves, $a$ is the width of the slit, and $n$ is a nonzero integer.

Limitations: This equation applies to a rectangular slit whose height is much greater than its width $a$. It is accurate only at points very distant from the slit compared to $a$. The slit must be oriented so that a given wave crest moves through all parts of the slit simultaneously.

The great majority of the diffracted wave's energy is contained in the central range $-\theta_{1 d}<\theta<\theta_{1 d}$ between the innermost angles of destructive interference: the wave amplitude in any of the fringes is much smaller than that in the central region.

## Section Q2.4: Optical Resolution

Light passing through a circular opening is diffracted into a bright central circle flanked by dimmer circular fringes, separated by dark rings where completely destructive interference occurs. The angle of the innermost ring of destructive interference in this case turns out to be

$$
\begin{equation*}
\sin \theta_{1 d}=1.22 \frac{\lambda}{a} \tag{Q2.13}
\end{equation*}
$$

This means that light from a point source will spread out into a diffraction pattern after going through an optical instrument's aperture, blurring the image somewhat (even if the instrument is perfectly focused). An instrument can resolve two point sources only if the angle $\theta$ between those sources is such that

$$
\begin{equation*}
\theta>\theta_{1 d} \approx \sin ^{-1} \frac{1.22 \lambda}{a} \tag{Q2.14}
\end{equation*}
$$

Purpose: This equation specifies the minimum angular separation $\theta$ that sources can have if they are to be resolved by an optical instrument whose aperture has diameter $a$.

Symbols: $\lambda$ is the average wavelength of the light from the source, and $\theta_{1 d}$ is the innermost angle of destructive interference.

Limitations: This equation is an approximation that becomes better if the instrument's sensor screen is far from the aperture compared to the aperture's diameter.

Note: Equation Q2.14 expresses what is called the Rayleigh criterion.

Instruments that can resolve sources at this limit are said to be diffraction-limited. Human eyes are very nearly diffraction-limited when operating in bright light.

## The phenomenon of diffraction



Figure Q2.1
The diffraction of water waves in a ripple tank. Line waves moving from the left disperse in a circular pattern after going through a small slit in a barrier.

A qualitative introduction to two-slit interference

Figure Q2.2
Circular waves emerging from two closely spaced slits will interfere with each other in the region where the waves overlap.

## Q2.1 Two-Slit Interference

In chapter Q1, we extensively explored the behavior of one-dimensional waves such as waves on a string or sound waves in a tube. In this chapter, we will consider the behavior of two-dimensional waves, such as waves on the surface of a body of water. Three-dimensional waves (such as sound waves) evaluated on a specified surface in space will behave in the same way.

Imagine a sinusoidal wave in water whose successive crests are parallel straight lines: we call such a two-dimensional wave a line wave. Imagine that such a line wave approaches a small gap (usually called a slit) in some kind of barrier. We empirically observe that waves traveling through a sufficiently small slit spread out to form essentially circular waves (i.e., waves whose successive crests are concentric circles) centered on the slit, as if the slit were a point source of waves (see figure Q2.1). This phenomenon is called diffraction.

One can intuitively see why this has to happen with water waves at least. As the wave moves through the slit, its sides are sheared off by the slit's sides. After the wave emerges from the slit, it will spread out to smooth out the sharp vertical edges where its sides used to be, creating the circular pattern. We will discuss diffraction in greater detail in section Q2.3: for our purposes now, it is enough to know that line waves going through a sufficiently small slit become circular waves.

With this in mind, imagine that we send line waves through two narrow slits a short distance apart, and assume that a given wave crest arrives at each slit simultaneously. The circular waves emerging from the slits will overlap each other, as shown in figure Q2.2. The superposition principle implies that the total wave at any given point will be the algebraic sum of the waves from each slit. At some points in the overlap region, wave crests from each slit will arrive at the same time, and the sum will be a wave with twice the amplitude of the waves from each slit: we say that the waves constructively interfere with each other at such a point. At certain other points, a crest from one slit will arrive at the same time as a trough from the other, and the sum of the waves will be zero: we say that the wave destructively interfere with each other at such a point. Constructive interference and destructive interference are illustrated in figure Q2.3.




(a)


$$
=\xrightarrow{1} t
$$

(b)

Figure Q2.3
(a) If at a given point, wave crests from the two slits arrive at the same time, the resulting wave at that point has twice the amplitude of the original waves: this is constructive interference. (b) If a wave crest from one slit arrives at the same time as the trough from the other, the resulting wave has zero amplitude: this is destructive interference.


Figure Q2.4
(a) If we measure the total wave amplitude $A$ along the line $B C$ and plot $A^{2}$ (a nonnegative measure of the total wave's strength) as a function of $y$, we get the graph shown (sideways) at the right edge. (b) A photograph of a two-slit interference pattern for real water waves showing the lines $x, a, a^{\prime}, b$, and $b^{\prime}$.

An examination of figure Q2.2 shows that the points on the water's surface where the waves interfere constructively lie on lines that appear to radiate from a point midway between the two slits (these lines are labeled $x, a, a^{\prime}, b, b^{\prime}$ in figure Q2.4). The points where the waves interfere destructively lie on similar lines. If we were to graph the wave amplitude along the line marked $B C$ on the diagram, we would find that points of constructive interference alternate with points of destructive interference, as shown in figure Q2.4a (the graph has been plotted sideways to make its connection with the diagram clear).

## Figure Q2.5

Assume that $P$ is a point where the waves interfere constructively. The extra distance that a wave from $S$ has to go to get to point $P$ compared to a wave from $Q$ is the distance from $S$ to $R$. This distance is roughly equal to $d \sin \theta$ (where $d$ is the distance between slits) if $P$ is very far away compared to $d$.

Determining the angles of maximal constructive " interference

The equation for the angles of maximal constructive interference in a two-slit interference experiment


We can use a simple geometric argument to determine the angles $\theta$ that the lines $x, a, a^{\prime}, b, b^{\prime}$, etc. make with the horizontal direction ( $x$ axis). Consider a distant point $P$ where the waves from each slit interfere constructively, and imagine lines connecting the point $P$ to slits $Q$ and $S$, as shown in figure Q2.5. Draw a line from slit $Q$ to a point $R$ on the line from $S$ to $P$ such that the distance between $R$ and $P$ is the same as the distance between $Q$ and $P$ (that is, $\triangle Q R P$ is an isosceles triangle). The extra distance that a wave from slit $S$ has to travel to get to $P$ is then the distance between points $S$ and $R$.

Now, if point $P$ is very far from the slits, lines $Q P$ and $S P$ make almost the same angle $\theta$ with the $x$ axis, the angle $\phi$ in the small triangle $\triangle Q R S$ is approximately equal to $\theta$, and the largest angle in that triangle is approximately a right angle. This means that if the distance between the two slits is $d$, the extra distance that the wave from slit $S$ has to cover to get to point $P$ is approximately equal to $d \sin \theta$.

Now, if $P$ is to be a point where the waves from $S$ and $Q$ interfere constructively, then the distance $d \sin \theta$ between $R$ and $S$ must be equal to an integer number of the wave's wavelengths, so that crests from $Q$ and $S$ arrive simultaneously at $P$. So the condition for constructive interference at point $P$ is simply

$$
\begin{equation*}
d \sin \theta_{n c}=n \lambda \quad \Rightarrow \quad \theta_{n c}=\sin ^{-1} \frac{n \lambda}{d} \tag{Q2.1}
\end{equation*}
$$

Purpose: The points at which waves emerging from two slits in a barrier constructively interfere most strongly form lines that radiate from the point halfway between the slits. This equation specifies the angles $\theta_{n c}$ that those lines make with the direction perpendicular to the barrier.

Symbols: $\lambda$ is the wavelength of the waves, $d$ is the center-to-center distance between the slits, and $n$ is an integer.

Limitations: This equation is accurate only at points very distant from the slits compared to $d$. The slits must be arranged so that a given crest of the line wave moves through both slits simultaneously. The slits need to be comparable in width to $\lambda$ to generate circular waves that are strong at all physically reasonable angles (see section Q2.3).

The value $n=0$ yields an angle $\theta_{0 c}=0$, which corresponds to the line of constructive interference along the $x$ axis in figure Q2.4. The values $n= \pm 1$ yield the angles $\theta_{1 c}$ and $\theta_{-1 c}$ of lines $a$ and $a^{\prime}$, the values $n= \pm 2$ yield the angles $\theta_{-2 c}$ and $\theta_{2 c}$ of the lines $b$ and $b^{\prime}$, and so on.

## Exercise Q2X. 1

Argue that in the limit that the large angles in the isosceles triangle $\triangle P Q R$ are roughly $90^{\circ}$ (that is, the angle $\angle Q P R \approx 0$ ), then $\theta \approx \phi$.

## Exercise Q2X. 2

If the distance between point $P$ and the slits is about 3 m and the distance $d$ between slits is 4 cm , then the large angles in triangle $\triangle P Q R$ are actually about $89.6^{\circ}$. What is the difference between $\theta$ and $\phi$ in this case?

## Example Q2.1 Interference of Water Waves

Problem Imagine that line water waves with a wavelength of 1.2 cm moving in the $+x$ direction go through two slits in a barrier parallel to the $y$ axis. The slits are separated by 4.0 cm (center to center). Imagine that we measure the wave amplitude along a line parallel to the barrier but 1.0 m from it. At what points along this line will the waves constructively interfere most strongly?

Translation The diagram below shows the situation and defines some useful symbols. Note that I have defined the midpoint between the slits to have coordinate $y=0$.


Model Since $D=1.0 \mathrm{~m} \gg 4.0 \mathrm{~cm}=d$, the condition on equation Q 2.1 is satisfied here. We can thus use that equation to find the angles $\theta_{n c}=$ $\sin ^{-1}(n \lambda / d)$ of lines along which maximal constructive interference occurs. Basic trigonometry implies that these lines intersect the line along which we measure the waves at positions $y_{n}=D \tan \theta_{n c}$. We know $D, \lambda$, and $d$, so we know enough to solve for $\theta_{n c}$ and $y_{n}$ for various values of $n$.

Solution The specific angles are

$$
\begin{array}{ll}
n=0: & \theta_{0 c}=0 \\
n= \pm 1: & \theta_{ \pm 1 c}=\sin ^{-1}\left(\frac{ \pm \lambda}{d}\right)=\sin ^{-1}\left(\frac{ \pm 1.2 \mathrm{~cm}}{4.0 \mathrm{~cm}}\right)=\sin ^{-1} 0.30= \pm 17.5^{\circ} \\
n= \pm 2: & \theta_{ \pm 2 c}=\sin ^{-1}\left(\frac{ \pm 2 \lambda}{d}\right)=\sin ^{-1}( \pm 0.60)= \pm 37^{\circ}  \tag{Q2.2}\\
n= \pm 3: & \theta_{ \pm 3 c}=\sin ^{-1}\left(\frac{ \pm 3 \lambda}{d}\right)=\sin ^{-1}( \pm 0.90)= \pm 64^{\circ}
\end{array}
$$

Values of $n$ such that $|n \lambda / d|>1.0$ yield no meaningful solution for $\theta_{n c}$, so the series stops here. The corresponding positions along the measurement line where the waves are strongest are

$$
\begin{array}{ll}
n=0: & y_{0}=D \tan \theta_{0 c}=0 \\
n= \pm 1: & y_{ \pm 1}=D \tan \theta_{ \pm 1 c}=(1.0 \mathrm{~m}) \tan \left( \pm 17.5^{\circ}\right)= \pm 0.32 \mathrm{~m}  \tag{Q2.3}\\
n= \pm 2: & y_{ \pm 2}=D \tan \theta_{ \pm 2 c}=(1.0 \mathrm{~m}) \tan \left( \pm 37^{\circ}\right)= \pm 0.75 \mathrm{~m} \\
n= \pm 3: & y_{ \pm 3}=D \tan \theta_{ \pm 3 c}=(1.0 \mathrm{~m}) \tan \left( \pm 64^{\circ}\right)= \pm 2.05 \mathrm{~m}
\end{array}
$$

Evaluation These results have the right units and are plausible.

While so far we have considered only water waves, the argument leading to equation Q2.1 applies to waves of all kinds, including three-dimensional waves (such as sound waves) as long as we evaluate the waves in a plane that contains the waves' direction of motion as well as both slits or sources, and as long as wave crests emerge from the slits or sources simultaneously.

## Exercise Q2X. 3

Imagine that you place your stereo speakers outside (to provide music for a party). They are placed distance of 1.5 m apart along a line we will call the $y$ axis. Imagine that both speakers are reproducing the sound of a female singer holding a solo high A $(880 \mathrm{~Hz})$ for several seconds. Consider a line parallel to the $y$ axis but 8.0 m from it. At what points along this second line would the sound be loudest? At what points would it be weakest?

## Q2.2 Two-Slit Interference of Light

Few physical phenomena are so common and important to our daily experience as light. Yet light is so subtle that understanding its nature has challenged the scientific community for thousands of years. Newton himself wrote a book on the subject (Opticks, 1704), but this book is not as famous as his work on mechanics because his particle model of light was seemingly contradicted by research done in the early 1800s by Young, Fresnel, Fitzeau, Lloyd, and Kirchhoff, which demonstrated in what seemed to be a fairly conclusive fashion that light was a wave. Maxwell's greatest triumph was his claim (later supported by experiment) that light was in fact an electromagnetic wave. This triumph, in combination with work in the late 1800s that established the frame independence of the speed of light, prompted Einstein to develop the theory of relativity. At roughly the same time, research into how atoms absorbed and emitted light was beginning to lay the foundations for quantum mechanics. The theory of quantum electrodynamics that eventually resulted served in turn as the template for those who developed quantum field theories for the weak and strong nuclear interactions in the 1970s. The study of light (and the challenges it raised) thus provided either the impetus or the template for essentially all the major revolutions in physics since the 1840s!

It is appropriate, therefore, that we start our investigations of quantum mechanics by studying light. My goal in this chapter is to bring your understanding of light up to where most physicists stood in the late 1800s, the eve of the quantum revolution. At this time, physicists were firmly convinced that light was a wave. What was the evidence supporting such a conclusion?


Figure Q2.6
(a) An opaque mask with two slits. (b) A top view of Young's two-slit interference experiment.


A Victorian physicist might have pointed to the two-slit interference experiments performed by Thomas Young during the first decade of the 1800s. Young directed a beam of light having essentially a single color (monochromatic light) on an opaque mask in which two narrow slits had been cut, as illustrated in figure Q2.6a. Light that passes through the slits falls on a screen placed some distance behind the mask, as illustrated in figure Q2.6b.

If light consisted of particles (as Newton thought), particles passing through the slits would travel in straight lines to the second screen, implying that we would see two isolated bright lines on the display screen, one for each slit, as shown in figure Q2.7a. If we take into account the fact that particles of light can follow slightly different straight paths from the source to the screen through each slit, we might expect the images of the slits be somewhat blurred; but there should be two and only two slit images, and a plot of the intensity of light as a function of distance $y$ along the display screen would look something that shown at the far right of figure Q2.7a.

If we do this experiment, though, we see not two, but many bright spots, as shown in figure Q2.7b. The spot spacing does depend on the slit separation,

Figure $\mathbf{Q 2 . 7}$
(a) The outcome of the Young double-slit experiment predicted by Newton's particle model of light. (b) The actual outcome of the Young double-slit experiment. (c) A photograph of an actual twoslit interference pattern.

Young's two-slit interference experiment

What a particle model of light predicts

The actual results

A wave model successfully explains these results
but reducing the slit separation counterintuitively increases the separation of the bright spots! We also find that the spot separation depends on the color of the light used, decreasing as one goes from red to violet.

These results are clearly incompatible with a simple particle model of light, but are easily explained by a wave model. If monochromatic light consists of waves having a well-defined wavelength, Young's two-slit light interference experiment is essentially the same as the two-slit water wave interference experiments considered in section Q2.1. We can interpret the bright and dark spots on the screen as positions where light waves interfere constructively or destructively, respectively. The interference pattern displayed in figure Q2.7b is qualitatively quite similar to the one for water waves shown in figure Q2.4, and quantitative measurements show that equation Q2.1 accurately describes how the positions of these peaks change as we vary the distance between slits. This experiment therefore offers compelling evidence that light is a wave.

What Is the Wavelength?
Problem Imagine that we have a laser that produces monochromatic red light of an unknown wavelength. We allow the laser's light to fall on a pair of slits whose center-to-center distance is $d=0.050 \mathrm{~mm}$, and we display the resulting interference pattern on a screen a distance $D=3.0 \mathrm{~m}$ from the slits. If the distance between adjacent spots on the screen has a very nearly constant value of $s=3.8 \mathrm{~cm}$ for spots near the central dot, what is the laser light's approximate wavelength?

Translation The drawing below illustrates the experiment.


Model In equation Q2.1, the central maximum will correspond to $n=0$ (since $n=0$ implies $\theta=0$ ). The angle between the line connecting the slits with the central maximum and that connecting the slits and a bright spot at position $y_{n}$ is

$$
\begin{equation*}
\theta_{n c}=\tan ^{-1} \frac{y_{n}}{D} \approx \frac{y_{n}}{D} \tag{Q2.4}
\end{equation*}
$$

where I have used the fact that $\tan \theta \approx \theta$ for small angles measured in radians. Note that for the $n=1$ spot, the angle comes out to be $0.038 \mathrm{~m} / 3.0 \mathrm{~m}=$ $0.0127 \mathrm{rad}=0.7^{\circ}$, so the small-angle approximation will be justified in this case for spots with reasonably small $n$. For small angles we also have $\sin \theta \approx \theta$, so equation Q2.1 implies that

$$
\begin{equation*}
\frac{n \lambda}{d}=\sin \theta_{n c} \approx \theta_{n c} \approx \frac{y_{n}}{D} \tag{Q2.5}
\end{equation*}
$$

Note that in this approximation $y_{n} \propto n$, so the spots are evenly spaced. If we define $s \equiv y_{n+1}-y_{n}$, then equation Q2.5 implies that in the small-angle limit,

$$
\begin{equation*}
\frac{(n+1) \lambda}{d}-\frac{n \lambda}{d} \approx \frac{y_{n+1}}{D}-\frac{y_{n}}{D} \quad \Rightarrow \quad \frac{\lambda}{d} \approx \frac{s}{D} \tag{Q2.6}
\end{equation*}
$$

Since we know $d, s$, and $D$, we can solve for $\lambda$.
Solution Doing this, we find that

$$
\begin{equation*}
\lambda \approx \frac{s d}{D}=\frac{(0.038 \mathrm{mr})\left(0.05 \times 10^{-3} \mathrm{~m}\right)}{3.0 \mathrm{~m}}=6.3 \times 10^{-7} \mathrm{~m}=630 \mathrm{~nm} \tag{Q2.7}
\end{equation*}
$$

Evaluation Note that the units are correct. The small value of this wavelength helps explain why wave aspects of light are not immediately obvious.

Note how the small-angle approximation helps simplify the mathematics in this example. Many practical interference experiments with light involve small angles, so equation Q2.6 is a variation of equation Q2.1 worth remembering.

Interference experiments provide a practical means of measuring the wavelength of light because they link $\lambda$ to quantities such as $d$ and $D$ that are big enough to measure with a ruler. Such experiments show that the wavelengths of visible light range from about 700 nm (deep red) to 400 nm (deep violet).

## Exercise Q2X. 4

Imagine that we replace the slits described in example Q2.2 with slits whose center-to-center spacing is 0.030 mm . Find the spacing between adjacent bright spots now. (Hint: You should find that the spacing gets larger.)

## Exercise Q2X. 5

Explain in words why the separation between the displayed bright spots increases when the separation between the slits decreases. Do not appeal to equations: rather, base your explanation on figure Q2.5.

## Exercise Q2X. 6

Imagine that we use a different laser in the experiment described in exercise Q2X.4, a laser that produces green light with a wavelength of 510 nm . What is the spacing between bright spots now?

## Q2.3 Diffraction

You may have noticed that not all the dots in the interference pattern shown in figure Q2.7c are equally bright. To explain why this is so, we need to better understand what happens to waves going through a single slit.

We can understand this most easily with the help of a simplified model of wave propagation that we call Huygens's principle (after the Dutch

Such experiments allow us to measure the wavelength of light

## Huygens's principle



Figure Q2.8
Propagation of a line wave according to Huygens's principle.

An analysis of what happens to a wave passing through a single slit

Figure Q2.9
(a) A top view of a single-slit diffraction experiment. (b) A magnified image of the slit region in this experiment. Here we treat the wave crest in the slit as if it were 12 points that emitted circular waves. Each circular wave contributes to the wave at point $P$.
physicist Christian Huygens who first proposed this model in 1678). This principle states

We can model each point on a given wave crest to be a point source of circular (or spherical) wavelets. At a time $\Delta t$ later, the new position of the wave's crest will be a curve (or surface) tangent to the wavelet crests.
This principle essentially expresses the idea that when each point in a medium is disturbed by the wave, the effects of that disturbance move radially away from that point as time passes. For example, if a bit of water is lifted above the surface of a pond by the crest of a wave, the water seeks to sink back downward; as it does, it pushes water on all sides outward, creating a circular ripple. The sum of all these circular ripples is what forms the crest as it moves forward.

Figure Q2.8 illustrates how this works for a line wave. We imagine each point on the line wave to be a point source for a circular wave (only a few points are shown for the sake of clarity). After time $\Delta t$, the wave from each point has expanded to a radius of $r=v \Delta t$, where $v$ is the speed at which waves travel in the medium in question. The tangent to these circular waves forms another straight line.

## Exercise Q2X. 7

Show, using the same approach, that the crest of an initially circular wave will spread out to form a larger circle as time passes.

Now consider the situation shown in figure Q2.9a. Imagine that line waves with a well-defined wavelength go through a slit of width $a$, and imagine that we measure the wave amplitude at various points $P$ along the $y$ axis, which is (at its closest) a distance $D \gg a$ from the slit. When $D$ is very large, the lines that waves must follow to get to $P$ from various points within the slit are approximately parallel lines, as shown in figure Q2.9b.

Now, according to Huygens's principle, we can imagine that every point along the crest of a wave passing through the slit emits a circular wave. To keep things relatively simple, we consider in figure Q2.9b only 12 points equally spaced along the slit.
(a)


When $\theta=0$ ( $P$ is directly in the $+x$ direction relative to the slit), the distance from each point to the final destination at point $P$ is equal, so the wavelets from each point interfere constructively when they reach $P$. But as we move our measurement point $P$ up the $y$ axis, the angle $\theta$ increases, and the distance that the wavelets have to move from each point to get to point $P$ is no longer the same. In particular, if the distance between a given pair of points in the slit is $d$, then the extra distance $s$ that a wavelet from the lower point has to go to get to $P$ beyond the distance to $P$ from the upper point is

$$
\begin{equation*}
s=d \sin \theta \tag{Q2.8}
\end{equation*}
$$

Thus as $\theta$ increases, the wavelets from each point get more and more out of phase with each other, and thus less and less strongly reinforce each other. The total wave amplitude will thus decrease as we move point $P$ up the $y$ axis.

When we reach the angle where $a \sin \theta=\lambda$, something interesting happens. Note that the distance between point 1 and point 7 is $\frac{1}{2} a$, as is the distance between points 2 and 8 , points 3 and 9 , and so on. This means that the wavelet from point 7 will have to travel a distance $s=\frac{1}{2} a \sin \theta=\frac{1}{2} \lambda$ farther to get to $P$ than the wavelet from point 1 does, and similarly for points 8 and 2,9 and 3 , and so on. Since one-half of a wavelength from a wave crest is a trough, the crest from point 1 will be canceled by a trough from point 7 , the crest from point 2 by a trough from point 8 , and so on. In short, the wavelet from each point in the upper half of the slit is exactly canceled by that from a point on the bottom half! We will therefore detect nothing at the angle $\theta_{1 d}$ such that

$$
\begin{equation*}
a \sin \theta_{1 d}=\lambda \quad \Rightarrow \quad \theta_{1 d}=\sin ^{-1} \frac{\lambda}{a} \tag{Q2.9}
\end{equation*}
$$

(The subscript tells us that this is the first angle of complete destructive interference.)

As we move point $P$ still farther up the $y$ axis, we begin to see waves again at point $P$, but they are $m u c h$ weaker. For example, consider the angle such that $a \sin \theta=(3 / 2) \lambda$. In this case, the wavelet from point 1 cancels the wavelet from point 5 , since the distance between points 1 and 5 is $\frac{1}{3} a$, so $s=\frac{1}{3} a \sin \theta=\frac{1}{2} \lambda$ again (the condition for destructive interference). Similarly, the wavelet from point 2 cancels that from point 6 , the wavelet from point 3 cancels that from point 7, and the wavelet from point 4 cancels that from point 8 . This leaves points 9 through 12 contributing to the net wave at point $P$ at this angle. But even wavelets from these points do not completely reinforce each other: wavelets from point 9 are almost completely out of phase with those from point 12, so the net wave at point $P$ at this angle comes mainly from points 10 and 11 . Thus it has about $2 / 12=1 / 6$ of the amplitude of the wave at $\theta=0$, where all 12 points contribute strongly. ( A more exact calculation yields a ratio of $1 / 4.7$.) The point is that the wave is generally much smaller in amplitude for angles larger than $\theta_{1 d}$ than it was near $\theta=0$.

## Exercise Q2X. 8

Argue that the waves again cancel completely at the angle $\theta_{2 d}$ such that $\sin \theta_{2 d}=2 \lambda / a$. (Hint: The wavelets from each point again cancel in pairs. Which points cancel which points in this case?)

Indeed, you can argue in a similar way that we will have completely destructive interference at any angle satisfying

The equation describing the angles of completely destructive interference in single-slit diffraction

The wave model leads to results consistent with experiment

Figure Q2.10
(a) A plot of wave intensity versus $\sin \theta$ for the waves of wavelength $\lambda$ emerging from a single slit of width a. (Note that if $\theta$ is small, $\sin \theta \approx \theta$ : in such a case, this will essentially be a plot of intensity versus $\theta$.) (b) The actual diffraction pattern produced by light going through a narrow slit. [The spacing between fringes in this photograph is about half of that shown in the graph in part (a).]

$$
\begin{equation*}
\theta_{n d}=\sin ^{-1} \frac{n \lambda}{a} \tag{Q2.10}
\end{equation*}
$$

Purpose: The points at which waves emerging from a single rectangular slit completely cancel out lie along lines that radiate from the slit's center. This equation specifies the angles $\theta_{n d}$ between those lines and the direction of motion of the original waves moving through the slit.

Symbols: $\lambda$ is the wavelength of the waves, $a$ is the width of the slit, and $n$ is a nonzero integer.

Limitations: This equation applies to a rectangular slit whose height is much greater than its width $a$. It is accurate only at points very distant from the slit compared to $a$. The slit must be oriented so that a given wave crest moves through all parts of the slit simultaneously.

This equation looks very much like equation Q2.1 for two-slit interference. Note, however, that equation Q2.1 refers to two-slit interference, while equation Q2.10 describes single-slit diffraction. Even more important, equation Q2.1 specifies angles along which the waves constructively interfere, but equation Q2.10 describes angles along which the waves destructively interfere (as the subscripts on the angles indicate)! Be aware of these differences!

If the wave model of light is correct, then light waves moving through a narrow rectangular slit must behave as we have argued. The brightness of any point on the display screen will depend on the energy per unit time per unit area (i.e., the intensity) of the light at that spot, which is proportional to the square of the wave amplitude. Figure Q2.10a shows a graph of the predicted intensity of light versus $\sin \theta$ for light moving through a narrow slit.

Figure Q2.10b shows a photograph of light emerging from a single slit. We can see that this pattern agrees completely with the prediction of the wave model. Experiments concerning the diffraction of light done in the early 1800 s provided some of the crucial evidence that finally convinced the physics community that Young was correct about light being a wave.

Note that virtually all the light energy is contained within the angular range of $-\theta_{1 d}$ to $+\theta_{1 d}$ between the first angles of completely destructive interference. The bright regions flanking this central region (which are called fringes of the diffraction pattern) contain comparatively small amounts of energy.


## Example 02.3 Diffraction of Light

Problem Light with a wavelength $\lambda=633 \mathrm{~nm}$ goes through a narrow slit of width $a$ and is then projected on a screen $D=3.0 \mathrm{~m}$ from the slit. The central bright region displayed on the screen is $w=2.0 \mathrm{~cm}$ wide. What is $a$ ?

Model The total angle $\Delta \theta$ spanned by the central maximum goes from $-\theta_{1 d}$ to $+\theta_{1 d}$, where $\theta_{1 d}=\sin (\lambda / a)$ (see equation Q2.8). Since angles are going to be small here, we can again use the approximation $\sin \theta \approx \tan \theta \approx \theta$, so we get

$$
\begin{equation*}
\frac{w}{D}=2 \frac{\frac{1}{2} w}{D}=2 \tan \theta_{1 d} \approx 2 \theta_{1 d}=2 \sin ^{-1} \frac{\lambda}{a} \approx \frac{2 \lambda}{a} \tag{Q2.11}
\end{equation*}
$$

Since we know $w, D$, and $\lambda$, we can solve for $a$.
Solution: Doing this, we get

$$
\begin{equation*}
a \approx \frac{2 D \lambda}{w}=\frac{2(3.0 \mathrm{~m})\left(633 \times 10^{-9} \mathrm{~m}\right)}{0.020 \mathrm{~m}}=1.9 \times 10^{-4} \mathrm{~m}=0.19 \mathrm{~mm} \tag{Q2.12}
\end{equation*}
$$

Evaluation: The units are right, and the result seems reasonable for a narrow slit.

Now we can understand the variation in the brightness of spots in the two-slit interference pattern. Light emerging from each slit fans out according to the single-slit diffraction pattern. If the two slits are the same width and are close together compared to the distance to the screen on which the patterns are to be displayed, their single-slit interference patterns will almost exactly overlap on the screen. If there were no interference between the waves emerging from two slits, the displayed intensity pattern would be essentially that of a single slit, as displayed in figure Q2.11a. Since the brightness of a bright spot in the two-slit interference pattern will depend on how much light is there in the first place to constructively interfere, the brightness of such a spot will be modulated by how bright the light in the single-slit pattern is at that point, as illustrated in figure Q2.11b.

The phenomenon of diffraction also explains why you can hear someone talking around a corner even when you can't see the person. Equation Q2.10 also suggests that longer wavelengths will diffract more widely than shorter


Explaining the brightness variation in the two-slit pattern

Why you can hear someone around a corner

Figure Q2. 11
(a) A graph of wave intensity versus position for light emerging from a single slit of width $a$. (b) A graph of the wave intensity versus position for light emerging from two slits of width a separated by distance $d=5 a$. Since the light required to generate the two-slit interference pattern has to come from the light provided by each single slit, the single-slit pattern provides the maximum possible intensity for the two-slit pattern.

Diffraction by a circular aperture


Figure Q2. 12
The diffraction pattern created by light after it has gone through a small circular aperture.

Implications for resolution of optical instruments

Rayleigh's criterion for resolving point sources
wavelengths. Sound waves have wavelengths on the order of magnitude of 1 m , so when sound waves go through an open doorway (which is also about a meter wide), they diffract pretty well in all directions, but light (whose wavelength is much shorter) is hardly diffracted at all. This effect also explains why voices around a corner may sound "muffled": the higherfrequency components of sound that add sharpness and clarity to a voice may not diffract as well to your position as lower-frequency components do.

## Q2.4 Optical Resolution

Figure Q2.10 displays the diffraction pattern of light going through a rectangular slit that is much longer than its width $a$. It is more difficult to calculate what happens to light when it goes through a circular aperture, but figure Q2.12 shows the result: we see a bright circular central region flanked by weaker circular fringes. It turns out that the angle of the innermost dark ring where complete cancellation occurs is given by

$$
\begin{equation*}
\sin \theta_{1 d}=1.22 \frac{\lambda}{a} \tag{Q2.13}
\end{equation*}
$$

where $a$ in this case is the aperture's diameter. This angle is only a bit larger than predicted by equation Q2.9.

## Exercise Q2X. 9

A laser on the International Space Station emits green light of wavelength 510 nm from a hole 5.0 mm in diameter. When this beam reaches the ground 180 km below, what is the approximate diameter of the laser beam's central region?

The fact that light is diffracted by a small opening has important implications for the ability of an optical instrument (such as a telescope or an eye) to resolve two distant objects whose angular separation is small. For example, imagine looking at two point sources of light (perhaps two stars) separated by a small angle $\theta$, as shown in figure Q2.13. The light from each source is diffracted somewhat as it goes through your pupil. This means that even if your lens focuses the light as well as possible, the light from each source creates a small but spread-out diffraction pattern on your retina.

Figure Q2.14 shows what the resulting diffraction patterns might look like on your retina for different angular separations between two sources. One can just begin to see that the sources are separate objects in the case shown in figure Q2.14b, where the central maximum of one diffraction pattern overlaps the first first minimum of the other. Therefore, we can see the sources as separate only when their angular separation $\theta$ is such that

$$
\begin{equation*}
\theta>\theta_{1 d} \approx \sin ^{-1} \frac{1.22 \lambda}{a} \tag{Q2.14}
\end{equation*}
$$

Purpose: This equation specifies the minimum separation angle $\theta$ that sources can have if they are to be resolved by an optical instrument whose aperture has diameter $a$.


Figure Q2.13
If two stars are separated by a sufficiently large angle, then the diffraction patterns produced on the retina when the light from the stars goes through the pupil do not overlap very much, so the retina will register these images as being separate. (The "bumps" on the retina in this drawing are meant to indicate graphs of the light intensity versus position on the retina.) If, however, the angle becomes much smaller, the diffraction patterns will begin to overlap, causing the two stars to look like a single blob.

Symbols: $\lambda$ is the average wavelength of the light from the source, and $\theta_{1 d}$ is the innermost angle of destructive interference.

Limitations: This equation is an approximation that becomes better if the sensor screen is far from the aperture compared to the aperture's diameter.

Note: Equation Q2.14 expresses what is called the Rayleigh criterion.

We call an instrument good enough to resolve sources separated by this angle diffraction-limited. The resolving power of a real instrument may be worse because of other factors, but it cannot be better than this. Since our ancestors depended so much on their eyes for staying alive, evolution has given us eyes that operate pretty close to the diffraction limit in daylight. The Hubble telescope, by virtue of its position above the atmosphere, also operates at close to the diffraction limit of its aperture. (The resolution of ground-based telescopes is limited more by unavoidable turbulence in the earth's atmosphere.)


Figure Q2.14
What the diffraction patterns for two point sources look like as their angular separation decreases. In (b), the angular separation of the sources is such that the central maximum of one pattern overlaps the first minimum of the other pattern. In this case, the sources can be barely distinguished as being separate.

## Example 02.4 The Resolving Ability of the Human Eye

Problem At night, the pupil of a typical person's eye opens up to as wide as 8 mm . What would be the smallest possible angular separation between two stars in the sky that the human eye might be able to resolve?

Model In this case, the aperture that the light goes through is the pupil, so $a=0.008 \mathrm{~m}$. The eye is most sensitive to light whose wavelength is in the center of the visual range, so let's estimate that $\lambda \approx 550 \mathrm{~nm}$. If we assume that the eye is diffraction-limited, this gives us enough information to apply the Rayleigh criterion. Since the angle will be pretty small, we can also use the small-angle approximation $\sin \theta \approx \theta$.

Solution Equation Q2.13 implies that

$$
\begin{align*}
\theta_{\min } & \approx \sin ^{-1} \frac{1.2 \lambda}{a} \approx \frac{1.2 \lambda}{a} \approx \frac{1.2\left(550 \times 10^{-9} \mathrm{~m}\right)}{0.008 \mathrm{~m}}=8 \times 10^{-5}(\mathrm{rad}) \\
& =\left(8 \times 10^{-5} \mathrm{rad}\right)\left(\frac{360^{\varnothing}}{2 \pi \mathrm{rad}}\right)\left(\frac{3600 \mathrm{arc} \mathrm{~seconds}}{1^{\gamma}}\right) \approx 20 \text { arc seconds } \tag{Q2.15}
\end{align*}
$$

Evaluation I have expressed the answer to only one significant digit because the approximations we have made are not more accurate than that. For comparison, earth-based telescopes are capable of resolving stars on the order of 1 arc second apart, while the Hubble telescope can in principle resolve stars on the order of 0.1 arc second apart. ${ }^{+}$

## TWO-MINUTE PROBLEMS

Q2T. 1 Waves from two slits $S$ and $Q$ will destructively interfere and cancel at a point $P$ if the distance between $P$ and $S$ is larger than the distance between $Q$ and $S$ by
A. $\lambda$
B. $n \lambda$
C. $\frac{n \lambda}{2}$ (where $n$ is an integer)
D. $\quad\left(n+\frac{1}{2}\right) \lambda$ (where $n$ is an integer)
E. $\frac{\lambda}{4}$
F. Other (specify)

Q2T. 2 Consider a two-slit interference experiment like the one shown in figure Q2.6b. The distance between adjacent bright spots in the interference pattern on the screen (A) increases, (B) decreases, or (C) remains the same if
(a) The wavelength of the light increases.
(b) The spacing between the slits increases.
(c) The intensity of the light increases.
(d) The width of the slits increases.
(e) The distance between the slits and the screen increases.
(f) The value of $n$ increases (careful!).

Q2T.3 We have seen that Huygens's principle implies that a circular wave front will remain circular and a line wave front will remain linear as time passes. Imagine that at an instant of time we set up a wave front that is shaped like a square moving away from its
center. Huygens's principle implies that this wave front will also maintain its square shape as time passes, true ( T ) or false ( F ) ?

Q2T. 4 Imagine that sound waves with a certain definite wavelength flow through a partially opened sliding door. If the door is closed somewhat further (but not shut entirely), the angle through which the sound waves are diffracted
A. Decreases
B. Increases
C. Remains the same
D. Depends on quantities not specified (explain)

Q2T. 5 Imagine that sound waves with a certain definite wavelength flow through a partially opened sliding door. If the wavelength of the waves increases, the angle through which the sound waves are diffracted
A. Decreases
B. Increases
C. Remains the same
D. Depends on quantities not specified (explain)

Q2T. 6 Line waves with wavelength $\lambda$ going through a slit with width $a$ will be diffracted into circular waves with approximately equal amplitude in all forward directions
A. Always
B. Never
C. Only if $a \gg \lambda$
D. Only if $a \ll \lambda$
${ }^{\dagger}$ The measured visual acuity of the eye at night is actually more like 200 arcs . This is so because the dark-adapted eye averages the response of many retinal cells in order to be able to respond to very dim light. This averaging reduces visual acuity. In bright light, the human eye does . perform at close to the diffraction limit.

Q2T. 7
If the two slits in a two-slit interference experiment were so far apart that their diffraction patterns did not overlap, the pattern displayed would be consistent with a particle model of light, T or F?

Q2T. 8
Consider an experiment where we send monochromatic light to a distant screen through a single narrow slit. The distance between adjacent dark fringes in the diffraction pattern displayed on the screen (A) increases, (B) decreases, or (C) remains the same if
(a) The wavelength of the light increases.
(b) The intensity of the light increases.
(c) The width of the slit increases.
(d) The distance between the slit and the screen increases.
(e) We look at fringes farther from the central maximum.

Q2T. 9 In the region where their beams overlap, two car headlights will create a clear interference pattern on a distant screen, T or F ?

Q2T. 10
If we shine white light through two slits onto a distant screen, we will see a clear interference pattern on the screen, T or F ?

Q2T. 11 Evolution has given eagles and other predatory birds very sharp eyesight. A friend claims to have read that an eagle's eye has 10 times the resolution of a human eye in broad daylight. This is physically impossible, T or F ?

Q2T. 12 With an optically perfect 200-power telescope with a 1.5 -in.-diameter tube, you can resolve objects that are roughly how many times closer together than you could with your naked eye? (Choose the closest response, and ignore air turbulence.)
A. 500 times
B. 200 times
C. 100 times
D. 20 times
E. 5 times
F. No better at all

## HOMEWORK PROBLEMS

## Basic Skills

Q2B. 1 Water waves with an amplitude of 0.80 cm and a wavelength of 2.5 cm go through two openings in a barrier. Each opening is 1.2 cm wide, and the openings are separated by 12.0 cm (center to center). Find the angles of the lines along which the waves constructively interfere.

Q2B. 2
Sound waves with a frequency of 320 Hz are emitted by two speakers 44 cm wide and 3.5 m apart. Find the angles of the lines along which the waves from the speakers constructively interfere (assuming that wave crests are emitted by the speakers simultaneously).
Q2B. 3 Imagine that the distance between two slits in a given experiment is $d=0.050 \mathrm{~mm}$ and that the distance between the slit mask and the display screen is $D=1.5 \mathrm{~m}$. If the distance between adjacent interference bright spots (for low $n$ ) is about 2.0 cm on the screen, what is the wavelength of the light involved?

Q2B. 4 Imagine that the distance between two slits in a given experiment is $d=0.040 \mathrm{~mm}$ and that the distance between the slit mask and the display screen is $D=2.5 \mathrm{~m}$. If the distance between adjacent interference bright spots (for low $n$ ) is about 3.0 cm on the screen, what is the wavelength of the light involved?

Q2B. 5
Imagine that the light source for a two-slit interference experiment provides red light with a wavelength of 633 nm . If this light is sent through a pair
of slits separated by a distance $d=0.040 \mathrm{~mm}$, find the spacing between adjacent bright spots of the interference pattern when it is displayed on a screen 3.2 m from the slits.

Q2B. 6
Imagine that the light source for a two-slit interference experiment provides yellow light with a wavelength of 570 nm . If this light is sent through a pair of slits separated by a distance $d=0.030 \mathrm{~mm}$, show that the angle between the bright spot corresponding to $n=0$ and the bright spot corresponding to $n=1$ is about $1.1^{\circ}$. If the display screen is a distance $D=2.4 \mathrm{~m}$ from the slits, show that the distance between these bright spots on the screen is about 4.6 cm .

Q2B. 7 Explain in terms of wave concepts why sounds emitted from a person's mouth can be heard almost equally well in all directions.

Q2B. 8
As you are just about to round a corner, you hear two people talking some distance beyond the corner. One has a very deep voice, and the other has a high voice. Which voice more easily carries around the corner? Explain.

Q2B. 9 A stereo speaker 30 cm wide sounds a pure $1250-\mathrm{Hz}$ note. Within what angle from the forward direction will you be able to hear this note (in a perfectly absorbing room)?

Q2B. 10

Sound waves from a stereo inside a house go through a partially open sliding patio door to the yard outside. If the door opening were 12 cm wide,
what would be the lowest frequency of sounds that would not be diffracted in essentially all directions through that opening?

Q2B. 11 Ocean waves with an amplitude of 2.0 m and a wavelength of 15 m go through a $55-\mathrm{m}$ opening in a breakwater that is shaped like a line and protects a body of water shaped like a square 800 m on a side between the breakwater, the beach, and two rocky ridges on either side. Draw a sketch of this situation that accurately and quantitatively illustrates the angle through which the waves are diffracted. (Also show your work in computing that angle.) Indicate some points on your sketch where boats within the breakwater will feel little wave motion.

Q2B. 12 Light with a wavelength of 633 nm goes through a narrow slit. The angle between the first minimum on one side of the central maximum and the first minimum on the other side is $1.2^{\circ}$. What is the width of the slit?

Q2B. 13 Light of wavelength 441 nm goes through a narrow slit. On a screen 2.0 m away, the width of the central maximum of the diffraction pattern is 1.5 cm . What is the width of the slit?

Q2B. 14 The beam of light emitted from a certain laser has a wavelength of 633 nm and an initial diameter of 1.0 mm . What is the diameter of the beam when it reaches the moon ( $384,000 \mathrm{~km}$ away)?

## Synthetic

Q2S. 1 You are setting up a pair of PA speakers on a field in preparation for an outdoor event. Each speaker is 0.65 m wide, and the speakers are separated by 8.2 m . To test the speakers, your coworker plays a single tone through the speakers whose frequency is 440 Hz . You are standing 52 m directly in front of the speakers and facing them. Roughly how far would you have to walk to your left or right to hear the sound amplitude drop almost to zero? How much farther would you have to go to hear the amplitude go back to its original strength?

Q2S. 2 Two radio antennas 60 m apart broadcast a synchronized signal with a frequency of 100 MHz . Imagine that we have a detector 5.0 km from the antennas. At this distance, what is the separation between adjacent "bright spots" in the interference pattern along a line parallel to the line between the antennas?

Q2S.3 If we hold two flashlights parallel, will they create a clear interference pattern in the region where the two beams overlap? Carefully explain at least two reasons why not.

Q2S. 4 When you connect stereo speakers to an amplifier, it is important that the speakers be connected in
phase, so that if a signal from the amplifier pushes the cone of one speaker out at a given time, it pushes the cone of the other speaker out at the same time. Reversing the two wires connecting one of the speakers to the amplifier will make it so that the same signal pushes one speaker cone out but pulls the other one in. Explain why this could be a problem, or at least undesirable. Would you still be able to hear the music?

Q2S. 5 Two sets of sinusoidal line waves approach each other from opposite directions. These waves have exactly the same amplitude and wavelength. When they overlap, will they constructively interfere, cancel each other out, or do something else? Describe as carefully as you can what will happen when these waves meet.

Q2S. 6
How big a speaker would you need to create a directed beam of sound waves with a frequency of 440 Hz whose total width increases by only 5 m for every 100 m the beam goes forward?

Q2S. 7 Television sets (particularly older models) produce a "whistle" sound at about $15,800 \mathrm{~Hz}$ (this is the frequency with which the electron beam sweeps across the face of the screen). This sound is audible to most youngsters and adults who haven't lost their high-frequency hearing. Imagine that a TV set is on (with the "mute" on) in your sister's bedroom as you walk by in the hallway. If your sister's door is ajar, leaving an opening 6.0 cm wide, and your ear is about 1.5 m from the door as you walk by, for about how many centimeters of your walk will you be able to hear the TV "whistle" (if you can at all)?

Q2S.8 The two headlights on a certain approaching car are 1.4 m apart. At about what distance could you resolve them as being separate? Make appropriate estimates as needed.

Q2S. 9 Under ideal conditions and when Mars is closest, estimate the linear separation of two objects on Mars that can barely be resolved (a) by the naked eye and (b) the Hubble telescope (whose main mirror is 1 m in diameter).

Q2S. 10 Let's guess that a person can distinguish between letters of the alphabet, if he or she can resolve features of the letter roughly one-fourth the size of the letter. Let's assume that in bright light, a person's pupil is $\approx 3 \mathrm{~mm}$ in diameter, and that the eye is most sensitive to light with a wavelength of 550 nm . What is the approximate maximum distance that one could read letters that are 3 mm high? The letters in headings (TWO-MINUTE PROBLEMS, HOMEWORK PROBLEMS; exercise numbers, etc.) in this text are about this height: check your calculation by direct observation and report the results.

Q2S. 11 The paintings of Georges Seurat consist of closely spaced small dots ( $\approx 2 \mathrm{~mm}$ wide) of pure pigment. The illusion of blended colors occurs at least partly because the pupils of the observer's eyes diffract light entering them. Estimate the appropriate distance from which to view such a painting, considering the fact that art museums are usually very well lit. (Adapted from Serway, Physics, 3d ed., Saunders, Philadelphia, 1990, p. 1096.)

Q2S. 12 In J. R. R. Tolkien's The Lord of the Rings (volume 2, p. 32), Legolas the Elf claims to be able to accurately count horsemen and discern their hair color (yellow) 5 leagues away on a bright, sunny day. Make appropriate estimates and argue that Legolas must have very strange-looking eyes, have some means of nonvisual perception, or have made a lucky guess. (1 league $\approx 3.0 \mathrm{mi}$.)

## Rich-Context

Q2R. 1 You are at sea on a foggy night. You are trying to find out how far you are from the shore, but the fog is too thick to see anything. After a certain time of aimless sailing, you dimly hear two separate foghorns on your port (left) side, perpendicular to your direction of travel. Each foghorn emits one short blast of sound at a pitch of 120 Hz every 2.00 s exactly, and each sounds about as loud as the other. Looking at your map, you see two foghorn locations marked plausibly near what you guess your location to be, and on the map the foghorns are 1700 m apart, flanking the entrance to a harbor. At a certain time, you notice that you hear the blasts from the foghorns simultaneously. After you have sailed at a steady heading for 22 min at a speed of $8.8 \mathrm{~km} / \mathrm{h}$ (as measured by your boat's speedometer), you hear the foghorns exactly out of phase (one honks, then the other, then the first, etc.). Roughly how far are you from the foghorns now? (Hint: Treat this as a two-slit interference problem.)

Q2R. 2 You are the prosecuting attorney in a case. You are eliciting testimony from your star witness, who has said that he was sitting on his porch 600 ft away from the crime scene when he saw the defendant commit the crime in broad daylight. You ask, "How did you know that it was the defendant?" Reply: "I recognized the Angels baseball cap the defendant often wears, the same cap the defendant is wearing now." Question: "How did you know that it was this Angels cap and not an ordinary cap of the same color?" Reply: "I could see the big A very clearly." Question: "You are absolutely sure of this?" Reply: "Yes, absolutely." With sudden shock, you realize that your star witness is lying. How do you know this?

Q2R. 3 The phenomenon of refraction arises because light travels at different speeds in different media. The
speed of light in a vacuum is $c=299,792,458 \mathrm{~m} / \mathrm{s}$ (by definition of the meter). Light moves about 0.03 percent slower than this in air, 25 percent slower in water, 34 percent to 40 percent slower in glass, and so on. Physicists typically express the speed of light in a certain medium in terms of the medium's index of refraction, which we define to be
$n_{m}=\frac{c}{v_{m}}$
where $c$ is the speed of light in a vacuum and $v_{m}$ is its speed in the medium. The index of refraction for air is 1.0003 , water is 1.333 , glass is 1.5 to 1.6 , and so on. (Note that $n_{m}$ is not necessarily an integer and is always > 1.)

Figure Q2.15 shows plane waves of light crossing the boundary between two transparent media. Let us say that the speed of light in the upper medium is $v_{1}$ but $v_{2}<v_{1}$ in the lower medium. The plane waves approach the boundary in such a way that their direction of travel makes an angle of $\theta_{1}$ with the respect to a line perpendicular to the boundary between the surfaces. Let us focus our attention on a certain wave crest that at $t=0$ is just hitting the boundary at point $A$. The wave crest at point $B$ will hit the boundary at point $C$ a certain time $\Delta t$ later such that $v_{1} \Delta t$ is equal to the distance between points $B$ and $C$. In the meantime, the Huygens wavelet from point $A$ will have moved outward a distance of $v_{2} \Delta t$ in the lower medium. The wave front must therefore go through point $C$ and be tangent to the wavelet from point $A$. You can see from the diagram that this wave front after time $\Delta t$ has been twisted somewhat from its original direction: the wave front's direction of motion now makes a smaller angle $\theta_{2}$ with respect to the direction perpendicular to the boundary. Refraction is this bending of the direction of the motion of the light wave as it moves across the boundary.


Figure 02.15
A wave crest being refracted as it moves from a medium where its speed is $v_{1}$ to one where its speed is $v_{2}$. The arrows indicate the crest's direction of motion.

